BALAJI INSTITUTE OF I.T AND MANAGEMENT KADAPA

OPERATIONS RESEARCH
(17E00205)

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Units covered: **1st, 2nd & half of 3rd Units**

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Unit - I
Introduction to OR

* Meaning.
* Nature
* Scope and significance of OR.
* Typical Applications of OR.

The Linear programming problem :-

* Introduction.
* Formulation of linear programming problem.
* Findings of LP.
* graphical solution to LPP.
* simplex Method.
* Artificial variable techniques.
* Two phase Method.
* Variants of the simplex Method.

UNIT - 1

Introduction :-

The term 'operation research' was coined in 1940 by Mcclosky and Trefthen in a small town of a Bowdsey in England. it is a science that came into existence in a military context. During world war II, the military management of UK called on scientists from various disciplanes and organized them into teams to assist it in solving stratagic and tactical problems relating to air and land defence of the country. they were required to formulate specific proposals and plans for aiding the military commands to arrive at decesions on optimal utilization of scarce military resources and efforts and also to implement the decesion effectively. This new approach to the systematic and scientific study of the operations of the system was called operations Research (or) or, operation research. Hence OR can be termed as 'an art of winning war without actually fighting it'.

Definations :-

operation research has been defined in various ways and it is perhaps still too young to be defined in some authoritative way.

They have not been any uniformly acceptable defination of it as yet. some prominent ones proposed thus far are given below. There have also been developing with the development of the subject.

operation Research is a scientific method of providing executive departments with a quantitative basis for decesion regarding the operations under their control — ~~Mose~~ Morse and Kimbol (1946)

operation research is the scientific method of providing executive with an analytical and objective basis for decesions. — P.M.S Blackett (1948)

operation research is the art of giving bad answer to problems to which otherwise worse answers are given. — T.L saaty (1958)

operation research is a systematic method oriented study of the basic structure, characterstics functions and relationships of an organization to provide the executive with a sound, scientific and quantitative basis for decesion making

— E.L Arnoff & M.J. Netzorg.

operation research is a scientific approach to problem solving for executive management

— H.M Wagner.

operation Research is an aid for the executive in making his decesions by providing him with the quantitative information based on the scientific method of analysis — C. kittee

operation research is the scientific knowledge through interdisciplanary team effort for the purpose of determining the best utilization of limited resources. — H.A Taha.

The various definations given above bring out of the following essential characterstics of operations research :

i) system orientation ii) use of interdisciplanary term
iii) Application of scientific method.
iv) uncovering new problems.
v) Quantitative solutions.
vi) Human factors.

Scope of operations Research :-

There is a great scope for economists, statisticians, administrators and technicians working as a term to solve problems of defence by using the OR approach. Beside this, OR is useful in various other importent fields like :

i, Agriculture
ii, Finance
iii, Industry
iv, Marketing
v, Personnel Management
vi, Production Management
vii, Research and Development.

Phases of operation Research :-

The Procedure to be followed in the study of OR generally involves the following major phrases.

i, Formulating the problem
ii, constructing a mathematical model
iii, Deriving the solution from the model
iv, Testing the model and its solution (updating the model)
v, controlling the solution.
vi, Implementation.

Models in operation Research :-

A model in OR is a simplified representation of an operation, or is a Process in which only the basic aspects or the most important features of a typical problem under investigation are considered. The objective of a model is to identify significant factors and interrelationships. The reliability of the solution obtained from a model depends on the validity of the model representing the real system.

A good model must possess the following characterstics :

i) It should be capable of taking in to account, new formulation without having any change in its frame.

ii) Assumptions made in the model should be as few as possible

iii) variables used in the model must be less in number ensuring that it is simple and coherent.

iv) It should be open to Parametric type of treatment.

v) It should not take much time in the its constructions for any problem.

Advantages of a Model :-

There are certain significant advantages of using a model. These are :

1) Problems under consideration become controllable through a model.

2) A model Provides a logical and systematic approach to the Problem.

3) A model clearly shows the limitations and scope of an activity.

iv, It helps in finding useful tools the eliminate duplication of methods applied to solve problem.

v, It helps in finding solutions for research and improvement in system.

vi, It provides an economic description and explanation of eiether the operations, or the system it represents.

characteristics of a Good model :-

i, the number of variables used should be as few as possible

ii, the number of assumptions should be as few as possible

iii, It should be easy and economical to construct.

iv, It should assimilate the system environmental changes without change in the framework.

v, It should be adaptable to Parametric type of treatment.

classification of Models :-

classification of models is a subjective problem. Models may be distinguished.

1, By degree of abstraction

2, By function

3, By structure

4, By nature of an Environment

5, By the extent of generality.

Models by function :-

These models can further be classified as

i, Descriptive models

ii, Predictive models

iii, Normative Models.

i, Descriptive models :- They describe and Predict facts and relationships among the various activities of the Problem. They do not have an objective function as a Part of the model to evaluate decesion alternatives. Through them it is Possible to get the information no how one or more factors change as a result of changes in other factor.

ii, Normative or optimization models :-
They are Prescriptive in nature and develop objective decesion-rule for optimum solutions.

Models by structure :-

These models are represented by

i, Iconic models

ii, Analogue models and

iii, symbolic models.

i, Iconic or physical Models :-

These are pictorial representations of real systems and have the appearance of the real thing. An Iconoic model is said to scaled down or scaled up according to the dimensions of the model, which may be smaller or greater that that of the real item, e.g.:- city maps, blue prints of houses, globe and so on. These models are easy to observe and describe, but are difficult to manipulate and are not very useful for the purposes of prediction.

ii, Analogue models :-

These are more abstract than the iconic model as there is no similarity between these models and real life items. The models in which one set of properties is used to represent another set of properties are called analogue models. After the problem is solved, the solution is reinterpreted in terms of the original system. These models are less specific and concrete, but easier to manipulate than iconic models.

iii, Mathematic or symbolic models :-

These are most abstract in nature and employ a set of mathematical symbols to represent the components of the real system. These variables are related together by means of mathematical equations to describe the behaviour of the system. The solution of the Problem is then obtained by applying well-developed mathematical techniques to the model.

The symbolic model is usefully the easiest to manipulate experimentally and it is also the most general and abstract. Its function is more explanatory than descriptive.

Models by nature of an Environment :-

These models can be classified in to

i) Deterministic models and

ii) Probabilistic models.

i) Deterministic models :-

In these models are all Parameters and functional relationships are assumed to be known with certainity when the decesion is to be made. Linear Programming and break even models are good examples of deterministic models.

ii) Probabilistic (or) stochastic Models :-

These models have at least one Parameter or decision variable as a random variable. These models reflect to some extent the complexity of the real world and the uncertainity surrounding it.

Models by the Extent of Generality :-

These models can be categorized as.

i) specific models and

ii) General Models.

when a model presents a system at some specific time, it is known as specific model. In these models if the time factor is not considered; they are termed as static models. An inventory ~~models~~ problem of determining economic order quantity for the next period assuming that the demand in planning period would remain the same as the that of today is an example of static model. Dynamic Programing may be considered as an Example of dynamic model.

Simulation and Heuristic models fall under the category of general models. These models are used to explore alternative stratagies which have been overlooked Previously.

Uses and Limitations of operation Research :-

Uses :-

i) It Provides a logical and systematic approach to the problem.

ii) It allows modifications of Mathematical solution before they are put to use.

iii) suggests all the alternative courses of action for the same management

iv) Helps in finding avenues for new research and improvement in systems.

v) facilities improved quality of decesion.

vi) It makes the overall structure of the production problem more comprehensible and helps in dealing with the problem as a whole.

vii) Aids in preparation of future managers by improving their knowledge and skill.

viii) Aids in preparation of future managers by improving skills

ix) leads to optimum use of manager's production factor.

x) Indicates the scope as well as limitations of a problem.

Limitations :-

Models are only idealized specialization (or) representations of reality and cannot be regarded as absolute in any case.

The validity of model, for a particular situations, can be ascertained only by conducting experiments on it.

Mathematical model are applicable to only any specific categories of problem as they do not take qualitative factors in to account. All influence factors, which cannot be quantified, find no place in mathematical models.

operation research requires huge calculations which cannot be handled manually and require computers, resulting in heavy costs.

As it is a new field, there is a resistance from the employees to the new proposals.

The implimentations of OR mainly depends on the person (OR input) who Provides the solution, and the person (manager) who uses the solution.

Operations - Research And Decesion - making :-

operation Research or management science, as the name suggests, it is the science of managing, which most of the time is about ~~takt~~ making decisions. it is thus a decision science that helps the management to make 'better decisions', a pivotal word in managing.

Decesion-making can be improved and in fact there is a wide scope for such improvements. The essential charecterstics of all decesions are:

i, objectives ii, alternatives iii, Influencing factors

once these characteristics are known, we can work towards improving the decesions.
In OR, scientific quantification is used in order to make better management decesion.
Thus, in OR, the essential features of decisions, namely, objectives, alternatives and influencing factors, are expressed in terms of specific scientific quantifications or mathematical equations.
operation research helps to overcome the complexity of the decision-making mode as it provides the management with the much needed tools for improving their-decesions.

Unit - I

Linear Programming Problem

Procedure of formulating linear Programming Problem (LPP) :-

* To write down the decision variables of the problem.
* To formulate the objective function to be optimized (Maximization/minimization) as a linear functions of the decision variables.
* To formulate the others conditions of the Problem such as resource limitations. market constraints, inter relation b/w variables etc.
* To add the non-negativity constraints from the consideration. so that the negative values of the decision variables do not have any valued Physical interpretations.

Note :-

construct Decision variables

construct the objective function $\left\langle \begin{array}{l} \text{Max} \\ \text{min} \end{array} \right.$

Developing the subjective constraints

constructs the non-negativity constraints.

Problems

1, A Manufacture Produces two types of models M_1 and M_2 each model of M_1 requires 4hrs of grinding and 2 hrs of Polishing. where each model of M_2 requires 2 hrs and grinding and 5hrs of Polishing. The manufacture has 2 grinders and 3 Polishers each grinding works 40hrs in a week and each Polishing works 60hrs in a week. Profit of M_1 model is Rs.3 and profit of M_2 model is Rs.4 what ever produced in a week sold in the market. How should the manufacture allocates its production capacity of the 2 types of the models M_1 and M_2. show that he was make the minimum Profit in a week?

sol:

	Model (x_1) M_1	Model M_2 (x_2)	Availability
Grinding	4	2	40hrs ⇒ 20×40=80hrs
Polishing	2	5	60hrs ⇒ 3×60=180hrs
Profits	3	4	

Total grinding hours in a week in 80hrs.

Total Polishing hours in a week is 180hrs.

1> Decesion variables :-

Let us consider get x_1, x_2 are the decesion variables of models M_1 and M_2.

2> Objective function :-

Model M_1 function sold in the Market manufacture gate in Rs.3 of Profit.

Model M_2 function sold in the market manufacture gate in Rs.4 of profit.

$$\text{Max } Z = 3x_1 + 4x_2$$

3> Subject to constraints :-

a> Subject to constraint for grinding :-

* In the grinding model M_1 has 4hrs and Model M_2 has 2hrs but the manufacture has 2 grinders.

* Each grinder works in a week is 40hrs ⇒ $2 \times 40 = 80$hrs

$$4x_1 + 2x_2 \leq 80$$

b) Subject to constraints for Polishing :-

* In Polishing M_1 works 2hrs and M_2 5hrs works. But the manufacture has 3 Polishers.
* Each Polisher works in a week is 60hrs ⇒ 3×60hrs = 180hrs

$$2x_1+5x_2 \le 180.$$

4) Non-negativity constraints :-

$$x_1, x_2 \ge 0.$$

overall LPP is Max $Z = 3x_1+4x_2$

Subject to constraints $4x_1+2x_2 \le 80$

$2x_1+5x_2 \le 180$ and

$x_1, x_2 \ge 0.$

2) A firm uses lathes is Engaged to Producing 2 Products P_1 and P_2 each unit of Product of P_1 requires 2kg of rawmaterial and 4 labour hrs Processing where as each unit of Product of P_2 required 5kg of rawmaterial and 3 labour hrs of the same time. Every week. The firm was availability of 50kg of raw material and 60 labour hours. one unit of Product P_1 sold and Earn the Profit Rs.20 and unit of Product of P_2

sold and the earn the profit Rs.30 formulate this problem as linear programming to determine how many units of each of the product should be produce for week. so that the firm can earn maximumprofit, assume all units produced can sold in the market ?

Sol:-

	Product P_1 (x_1)	Product P_2 (x_2)	Availability
Raw material	2	5	50kg
labour hrs	4	3	60 labour hrs
Profit	20	30	

1) Decesion variables :-

Let us consider P_1 and P_2 are the decesion variables.

2) objective function :-

The manufacture Produced from Product P_1 can earn the profit Rs. 20/- and product P_2 can the profit Rs.30/-

$$\text{Max } Z = 20x_1 + 30x_2.$$

3) Subject to constraints :-

a) Subject to constraint for raw material :-

The Availability of a rawmaterial in a week is 50kg.

Product P_1 requires 2kg of rawmaterial and Product P_2 requires 5kg of rawmaterial

Rawmaterial subject to constraints is

$$2x_1 + 5x_2 \leq 50.$$

b) Subject to constraint for labour :-

The Availability of a labour in working hrs in a week is 60hrs.

Product P_1 requires 4 labour hrs and product P_2 3 labour hrs.

The subject to constraint on labour is

$$4x_1 + 3x_2 \leq 60.$$

4) Non-negativity constraints :-

$$x_1, x_2 \geq 0$$

overall LPP is Max $Z = 20x_1 + 30x_2$

subject to constraints $2x_1 + 5x_2 \leq 50.$

$$4x_1 + 3x_2 \leq 60$$

$$x_1, x_2 \geq 0$$

3) A company manufactures 2 produces are A and B. These products are processed in the same machine it takes 10mts in process 1 unit of product 'A' and 2mts for each unit of product 'B' and the machine operates for maximum 35 hrs in a week. product 'A' requires 1kg and 'B' requires 0.5kg of raw material for unit the supply of which is 600kg for week market constraint on product 'B' is known to be minimum 800units every week. product 'A' cost Rs.5/- per unit and solded Rs.10/- product 'B' cost Rs.6/- per unit price of Rs.8/-. Determine the no. of units of A and B for week to maximize the profit.

sol:

constraints	Products A (x_1)	B (x_2)	Availability.
Time.	10min	0.2min	35x60 = 2100
Raw material	1kg	0.5kg	600kg
Market	0	1	800units
Profit	5	2	

Profit on product A = sold cost - production cost

= 10 - 5

= 5

profit on product B = soldcost − production cost

$= 8 - 6$

$= 2$

1) Decesion variables :-

Let us consider x_1, x_2 are the decesion variables of product A and B

2) objective function :-

Here, profit of product 'A' Rs.5/- and product 'B' is Rs.2/-

$$Max\ z = 5x_1 + 2x_2.$$

3) Subject to constraints :-

a) Subject to constraints for Time :-

Here, The Availability of time in a week is 35hrs but the time constraint can be given as mts.

So $35 \times 60 = 2100$ mts.

The Time constraint for product 'A' is 10mts and product 'B' is 2mts.

Time constraint for product 'A' is 10mts and product B is 2mts.

Time constraint is $10x_1 + 2x_2 \leq 2100$

b) Subject to constraints for Rawmaterial :-

Here, The Availability of Rawmaterial in a week is 600kg.

The Rawmaterial of product 'A' is 1kg and 'B' is 0.5kg.

Rawmaterial constraints is $x_1 + 0.5x_2 \leq 600$.

c) Subject to constraints for Market :-

The Availability of Rawmaterial can be use a minimum of 800units.

Market constraints of product 'B' is 1 unit and 'A' is '0' units.

Market constraints $x_2 \geq 800$.

4) Non-negativity constraints :-

$x_1, x_2 \geq 0$.

over all LPP $Max\ z = 5x_1 + 2x_2$

Subject constraint to $10x_1 + 2x_2 \leq 2100$

$x_1 + 0.5x_2 \leq 600$

$x_2 \geq 500$ and

$x_1, x_2 \geq 0$.

4) A person requires 10,12,12 units of chemicals A, B and c respectively for his Jar a liquid product contains 5, 2 and 1 unit of A, B, c respectively. for Jar A dry product contains 1, 2 and 4 unit of A, B, C per carton. if the liquid product is sold Rs. 3/- per Jar and the dry product is sold for Rs. 2/- per carton, How many units of each product should been Purchased in order to minimize the cost and meet the requirements.

sol:-

constraints	Products.		Availability
	Jar (liquid) x_1	carton (solid) x_2	
A	5	1	10
B	2	2	12
C	1	4	12
solid (mini. cost)	3	2	

1) Decesion variables :-

Let x_1, x_2 decesion variables are Jar and carton.

2) objective function :-

Based on the problem consideration. we have to find to the minimum cost so that the objective function can be existed in maximize objective function can be sold at Rs.3/- per Jar and Rs.2/- per carton.

3, subject constraints :-

a, subject constraint 'A' :-

Both Jar and carton Here, chemical 'A' filled in both Jar and carton 5 and 1 units.

Then the Availability chemical both in the Jar and carton is 10 units.

$$Max\ Z = 5x_1 + 1x_2 \leq 10$$

b, subject constraint 'B' :-

Here, chemical 'B' filled in both Jar and carton 2 and 2 units. Then the availability chemical both in the Jar and carton in 12 units.

$$Max\ Z = 2x_1 + 2x_2 \leq 12.$$

c, Subject constraint 'c' :-

Here, chemical 'c' filled in both Jar and carton 1 and 4 units.

Then the Availability chemical both in the Jar and carton 12 units.

$$Max\ Z = 1x_1 + 4x_2 \leq 12.$$

5) A paper milk produces 2 grades paper namely x or Y owing to raw materials restructions. It can't produce more than 400 tones of grade X and 300 tones of grade Y in a week. There are 160 production hours in a week. it requires 0.2 and 0.4 hrs to produce a tone of products X and Y respectively with corresponding profit of Rs. 200 and Rs. 500 tone. formulate the above as an linear programming problem to maximize the profit and optimum product mix.

Sol:

constraints	Grade. (x_1)	Grade (x_2)	Availability
Rawmaterials x	1	0	400
y	0	1	300
production man hrs	0.2	0.4	160
Profits	Rs. 200	Rs. 500	

1) Decesion variables :-

Let us consider x_1, x_2 are Decesion variables of grade 'X' and grade 'Y'

2) objective function :-

Here, what ever the papers produced manufacturer grade x gets Rs. 200 profit and grade 'y' paper gets Rs. 500 profit.

$$\therefore \text{Max } Z = 200x_1 + 500x_2$$

3) subject to constraints :-

a) subject constraint for Raw material :-

Here, grade 'x' paper cannot produce more than 400 tones in a week.

$$x_1 \leq 400$$

Here, grade 'y' cannot produce more than 300 tones in a week.

$$x_2 \leq 300$$

b) subject constraint for Production man hrs :-

Grade x and Grade 'y' Availability of Production hrs in a week is 160.

grade 'x' production hrs is 0.2hr.

grade 'y' Production hrs is 0.4hrs in a week.

Then the subject to constraints Production hrs is $0.2x_1 + 0.4x_2 \leq 160$.

4) Non-negative constraints :-

$x_1, x_2 \geq 0$

overall LPP is $x_1 \leq 400$

$x_2 \leq 300$ subject to constraints &

Max $Z = 200x_1 + 500x_2$ and

$0.2x_1 + 0.4x_2 \leq 160$ & $x_1, x_2 \geq 0$.

6) A firm uses lathes, milling machines and grinding machines to produce 2 machine parts table given below represents the machining time require for each part, the machining times available on different machines and the profit on each machines profits.

Types of Machine.	Machine time requires for the mint. time. machine Parts (min)		Availability (min)
	I(x_1)	II(x_2)	
Lathes	12	6	3000
Milling machine	4	10	2000
grinding machine	2	3	900
Profit per unit	Rs. 40,	Rs. 200.	

Solve the formulate problem so that the no. of

Parts 1 and 2 to be manufacture for week. to Maximize the profit.

Sol:-

1) Decesion variables :-

Let us consider the decesion variables x_1, x_2 of machine parts I and II.

2) objective function :-

Here, machine Part I profit is Rs. 40 and Part II profit is Rs. 100.

$$\therefore \text{Max } Z = 40x_1 + 100x_2.$$

3) subject to constraints :-

a) subject to constraint for lathes :-

Here, Machine I contains the maximum 12 parts and having the lathes.

Machine II contains the maximum 6 parts and having the lathes.

$$12x_1 + 6x_2.$$

Millin.

b) subject to constraint for milling machine :-

Milling machine having the availability are 300.

Milling machine having $4x_1 + 10x_2$

Grinding machine having $2x_1 + 3x_2$.

Availability are 3000 and 2000.

For x_1 having 3000 min.

x_2 having 2000 min.

900 min.

4) Non-negativity :-

Non-negativity $x_1, x_2 \geq 0$.

overall LPP is Max $Z = 40x_1 + 100x_2$. and

subject to constraints

$$12x_1 + 6x_2 \leq 3000$$
$$4x_1 + 10x_2 \leq 2000.$$
$$2x_1 + 3x_2 \leq 900 \text{ and}$$
$$x_1, x_2 \geq 0.$$

7) A manufacturing unit has 3 products on their production line. The production capacity is 50, 30 and 45 respectively. The limitations in the production is that off 300man hrs as total availability and the manufacturing time required for product 0.5, 1.5 and 2.0 man hrs. The products are priced to result in profits Rs.10, Rs.20, Rs.15 respectively. If the company has a daily demand of 25 units, 20 units &

35 units for respective Products formulate the Problem as a LPP so as to maximize the Profit.

Sol:-

1, Decesion variables :-

Let us consider x_1, x_2, x_3 are the decesion variables of 3 products.

2, objective function :-

whatever the products produced by manufacturer he can get the profits. so the maximize objective function is appeared.

$$Max\ Z = 10x_1 + 15x_2 + 20x_3.$$

3, Subject to constraints :-

Here, Time constraint, Production capacity constraint and daily demand constraints are available.

1, Time constraint :- Here, The total availability of time for manufacturing the 3 products is 300hrs.

Time required to produce (or) to manufacture first product is 0.5hrs.

Time required to Produce second product is 1.5 hrs.

Time required to produce third product is 2.0 hrs and

The time constraint is

$$0.5x_1 + 1.5x_2 + 2.0x_3 \leq 300.$$

The production capacities subject to constraints are for first product production capacity is 50 and 2nd product production capacity is 30, 3rd product production capacity is 45.

i.e $x_1 \leq 50$

$x_2 \leq 30$

$x_3 \leq 45$

Daily Demand constraints :-

The daily demand constraints for the 1st product is 25 units.

2nd product is 20 units

3rd product is 35 units

$x_1 \geq 25$

$x_2 \geq 20$

$x_3 \geq 35$

Non-negativity constraints :-

$x_1, x_2, x_3 \geq 0$

Overall LPP is max $z = 10x_1 + 15x_2 + 20x_3$.

Subject to constraints

Time constraints $0.5x_1 + 1.5x_2 + 2.0x_3 \leq 300$.

production capacity $x_1 \leq 50$

$x_2 \leq 30$

$x_3 \leq 45$

Daily demand constraints $x_1 \geq 25$

$x_2 \geq 20$

$x_3 \geq 35$ and

$x_1, x_2, x_3 \geq 0$.

8) A financial advicer at delhi investments has identified 2 companies that are likely candidates for a take east-tern cable is the over in the near future. east-tern cable is a leading manufacturing of flexible cable systems used in the constructions industry and conswitch is a new firm (or) new industry specializing in digital switching system eastern cable is corrently trading Rs. 40 for share and conswitch is Rs. 25 for share. if the take over occurs the financial advicer estimates that the Price of Eastean cable will go to Rs. 55 for sale and conswitch will go to Rs. 43 for sale also it is found that conswitch. high risk sale. Assume that a client has indicated

a willingness to invest a maximum of Rs. 50,000 in the 2 companies the client wants to invest at least Rs. 10,000 in conswitch. due to high risk associated with conswitch, the financial advicer recommended that at most Rs. 25,000 should be Invested in conswitch.

a) Formulate a Linear Programming model for the investment decesion faced the client.

b) find the optimal solution through graphical method.

Sol:-

1) Decesion variables :-

Let us consider x_1, x_2 as the decesion variables of Eastern cable and conswitch.

2) objective function :-

Eastern cable is correntl trading Rs. 40 for share and conswitch is Rs. 25 for share. If the take over occurs the financial advicer Estimates that the price of Eastern cable will go to Rs. 55 Per share and conswitch will go to Rs. 43.

Profit/Loss of Eastern cable $= 55-40$
$= 15$

Profit/Loss of conswitch $= 43-25$
$= 18$.

$\text{Max } Z = 15x_1 + 18x_2$.

3) Subject to constraints :-

for the Eastern cable $x_1 + x_2 \leq 50,000$.

Eastern cable $x_1 \geq 15,000$
$x_2 \geq 10,000 \leq 25,000$.

4) Non-negativity constraints :-

$x_1, x_2 \geq 0$

overall LPP, $\text{Max } Z = 15x_1 + 18x_2$.

Subject to constraints $x_1 + x_2 \leq 50,000$
$x_1 \geq 15,000$
$x_2 \geq 10,000 \leq 25,000$.

so.., $x_1, x_2 \geq 0$.

Graphical method with LPP (Linear Programming Problem) :-

The Graphical model provides a pictorial representation of the solution process and a great deal of in sight in to the basic concept used in solving large linear programming problem.

Procedure for solving LPP in graphical method:-

The step involved in the graphical solutions are as follows.

Step-1 :- consider each inequality constraints in to the equations.

Step-2 :- plot each equation on the graph, as equation will Geometric represent a ~~statement~~ straight line.

Step-3 :- Mark the region, if the inequality contain corresponding to the line is $\leq$ then the region below the line lying in the first Quadrant is shared.

* For inequality constraints $\geq$ to sign the region above the line in the first Quadrent is shade.

* The point lying in the common region will satisfy all the constraints ~~simout~~ ~~as~~ simultaneously the common region that obtain the feasible region.

Step-4 :- Assign the orbitable value say zero to the objective fonction.

Step-5 :- Draw a straight line to represent the objective fonction with the orbitary value.

Step-6 :- stream the objective function line till the extreme point of the region.

Step-7 :- find the coordinates of the extreme point selected in step-6 and find the maximum (or) minimum value for the objective function.

Problems :-

1) solve the given LPP max $Z = 2x_1 + 3x_2$ subjective to constraints

$$x_1 + x_2 \leq 20$$
$$3x_1 + 2x_2 \geq 30$$
$$2x + 4x_2 \leq 40$$
$$x_1, x_2 \geq 0.$$

sol:- Let us consider the inequalities into the equations

$$x_1 + x_2 = 20 \longrightarrow (1)$$
$$3x_1 + 2x_2 = 30 \longrightarrow (2)$$
$$2x + 4x_2 = 40 \longrightarrow (3)$$

$x_1 + x_2 = 20$

Let $x_1 = 0$

Substitute $x_1 = 0$ in the equation

$0 + x_2 = 20$

$x_2 = 20$

$A(x_1, x_2) = (0, 20)$

Let $x_2 = 0$

$3x_1 + 2x_2 = 30$

Let $x_1 = 0$.

Substitute $x_1 = 0$ in the equation.

$3(0) + 2(x_2) = 30$

$0 + 2x_2 = 30$

$2x_2 = 30$

$x_2 = \frac{30}{2}$

$x_2 = 15$

$o(x_1, x_2) = (0, 13)$

Let $x_2 = 0$

Substitute $x_2 = 0$ in equation

$3x_1 + 2(0) = 30$

$3x_1 = 30$

$x_1 = \frac{30}{3}$

$x_1 = 10$

$2x_1 + 4x_2 = 40$

Let $x_1 = 0$ sobstitute the equation

$2(0) + 4x_2 = 40$

$4x_2 = 40$

$x_2 = 10$

$E(x_1, x_2) = (0, 0)$

Let $x_2 = 0$ sobstitute in the equation.

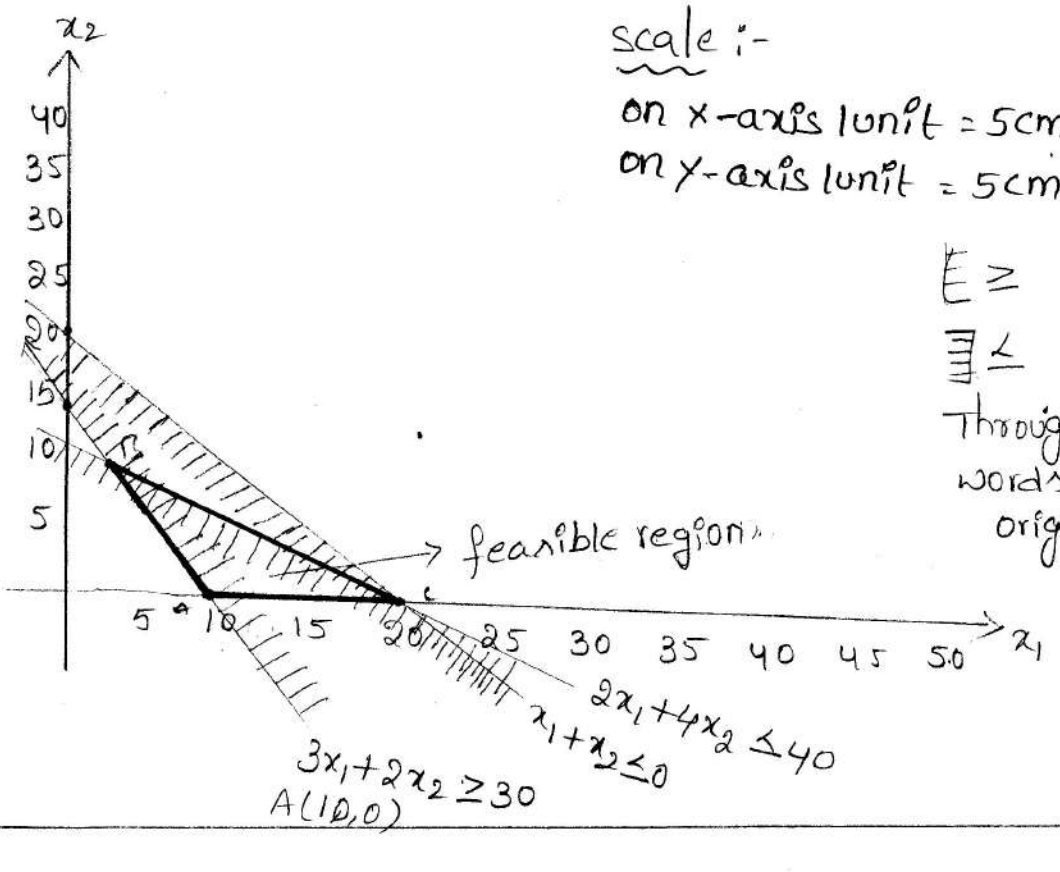
$2x_1 + 4(0) = 40$

$2x_1 = 40$

$x_1 = 20$

$f(x_1, x_2) = (20, 0)$

Plot the graph by using the coordinates are points.



Scale :-

on x-axis 1unit = 5cm

on y-axis 1unit = 5cm

E ≥

∃ ≤

Through words origin.

so 'c' intersects the line $3x_1+2x_2 \geq 30$ and $2x_1+4x_2=4$

$3x_1+2x_2=30 \longrightarrow (5)$

$2x_1+4x_2=40 \longrightarrow (6)$

$(5)\times(2) \Rightarrow \quad 6x_1+\cancel{4x_2}=60$

$\quad 2x_1+\cancel{4x_2}=40$

$4x_1=20$

$x_1=5$

substitute in equation (5)

$3x_1+2x_2=30$

$3(5)+2x_2=30$

$15+2x_2=30$

$2x_2=30-15$

$2x_2=15$

$x_2=15/2$

$x_2=7.5$

$\therefore$ The Extreme point $c(x_1,x_2)=c(5,7.5)$

$A(10,0)$, $B(20,0)$

corner point	objective function Max $Z = 2x_1+3x_2$
1, $A(10,0)$	$2(10)+3(0)=20+0=20$
2, $B(20,0)$	$2(20)+3(0)=40+0=40$
3, $C(5,7.5)$	$2(5)+3(7.5)=10+22.5=32.5$

$\therefore$ The point is $B(20,0)$ is the maximum value

$Max\ z = 40,\ x_1 = 20$ and $x_2 = 0$

2) A) Solve the following $Min\ z = 3x_1 + 2x_2$ subject to

$5x_1 + x_2 \le 10,\ 2x_1 + 2x_2 \le 12,\ x_1 + 4x_2 \le 12,\ x_1, x_2 \ge 0.$

sol:- Let us consider the inequality of the Inequations

$$5x_1 + x_2 = 10$$
$$2x_1 + 2x_2 = 12$$
$$x_1 + 4x_2 = 12$$

$5x_1 + x_2 = 10$

Let $x_1 = 0$ sobstitute in equation.

$5(0) + x_2 = 10$

$x_2 = 10$

$A(x_1, x_2) = (0, 10)$

Let $x_2 = 0$ substitute in equation.

$5x_1 + (0) = 10$

$5x_1 = 10$

$x_1 = 2$

$B(x_1, x_2) = (2, 0)$

$2x_1 + 2x_2 = 12$

Let $x_1 = 0$ sobstitute in equation.

$2(0) + 2x_2 = 12$

$$2x_2 = 12$$

$$x_2 = 6$$

$$c(x_1, x_2) = (0, 6)$$

Let $x_2 = 0$ substitute in equation.

$$2x_1 + 2(0) = 12$$

$$2x_1 = 12 \Rightarrow x_1 = 6$$

$$D(x_1, x_2) = (6, 0)$$

$$x_1 + 4x_2 = 12$$

Let $x_1 = 0$ substitute in equation.

$$0 + 4x_2 = 12$$

$$x_2 = 3$$

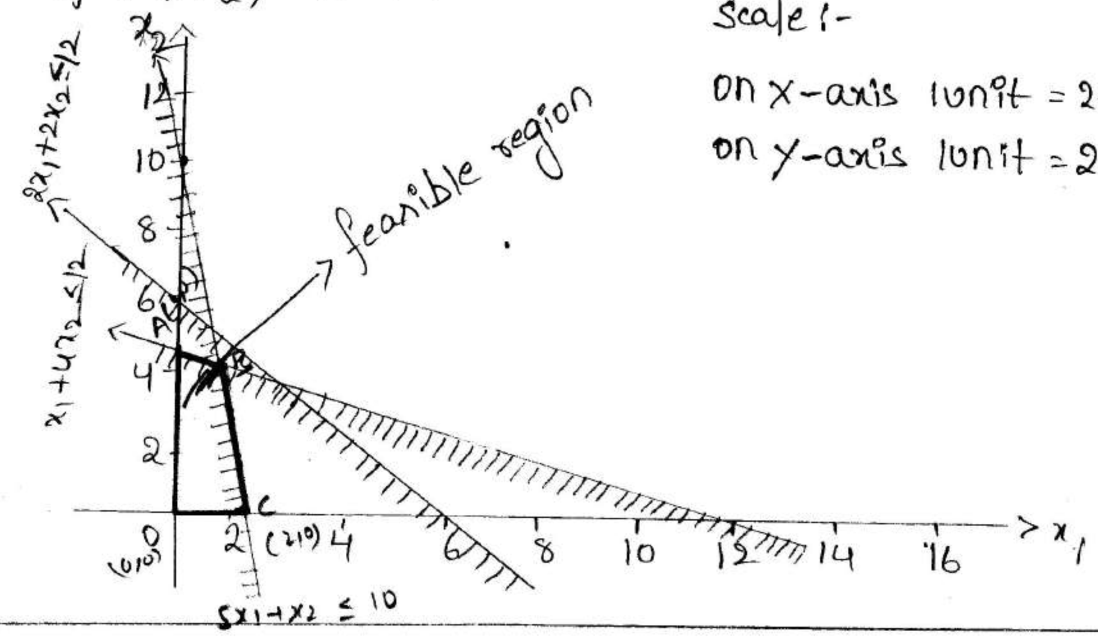
$$E(x_1, x_2) = (0, 3)$$

Let $x_2 = 0$ substitute in equation.

$$x_1 + 4(0) = 12$$

$$x_1 = 12$$

$$f(x_1, x_2) = (12, 0)$$



scale:-

on x-axis 1unit = 2cms

on y-axis 1unit = 2cms

The Point B intersect the line

$5x_1+x_2=10$ and $x_1+4x_2=12$

$5x_1+x_2=10 \longrightarrow (1)$

$x_1+4x_2=12 \longrightarrow (2)$

$$(1)\times 4 \Rightarrow 20x_1+\cancel{4x_2}=40$$

$$\underline{x_1+\cancel{4x_2}=12}$$

$$19x_1=28$$

$$x_1=\frac{28}{19}$$

$$\therefore x_1=1.5$$

substitute $x_1=1.5$ in the equation (2)

$$5x_1+x_2=10$$

$$5(1.5)+x_2=10$$

$$\Rightarrow 7.5+x_2=10$$

$$x_2=10-7.5$$

$$\therefore x_2=2.5$$

The Point $B(x_1,x_2)=(1.5,2.5)$

$O=(0,0)$, $A(2,0)$, $B(0,8)$

S.NO	corner point	objective function min $Z=3x_1+2x_2$ Min value
1	O(0,0)	$3(0)+2(0)=0+0=0$ - "
2	A(2,0)	$3(2)+2(0)=6+0=6$
3	B(1.5,2.5)	$3(1.5)+2(2.5)=4.5+5=9.5$ - max value
4	C(0,3)	$3(0)+2(3)=0+6=6$

∴ At point $O(0,0)$ the optional solution can be occured

$\min z = 0, \quad x_1 = 0$ and $x_2 = 0$

∴ At point $B(1.5, 2.5)$ the optional solution can be occur $\max z = 0.5, \quad x_1 = 1.5$ and $x_2 = 2.5$

B) Solve the following

$\text{Max } z = 20x_1 + 30x_2$

subject to $2x_1 + 5x_2 \leq 50$

$4x_1 + 3x_2 \leq 60$

$x_1 + x_2 \geq 0$

sol:- Let us consider the inequality of the inequation

$2x_1 + 5x_2 = 50$

$4x_1 + 3x_2 = 60$

$x_1 + x_2 = 0$

$2x_1 + 5x_2 = 50$

Let $x_1 = 0$ in equation

$2(0) + 5x_2 = 50$

$0 + 5x_2 = 50$

$x_2 = 10$

$A(x_1, x_2) = (0, 10)$

Let $x_2 = 0$ in equation

$2x_1 + 5(0) = 50$

$2x_1 = 50$

$x_1 = 25$

$B(x_1, x_2) = (25, 0)$

$4x_1 + 3x_2 = 60$

Let $x_1 = 0$ in equation.

$4(0) + 3x_2 = 60$

$3x_2 = 60$

$x_2 = 20$

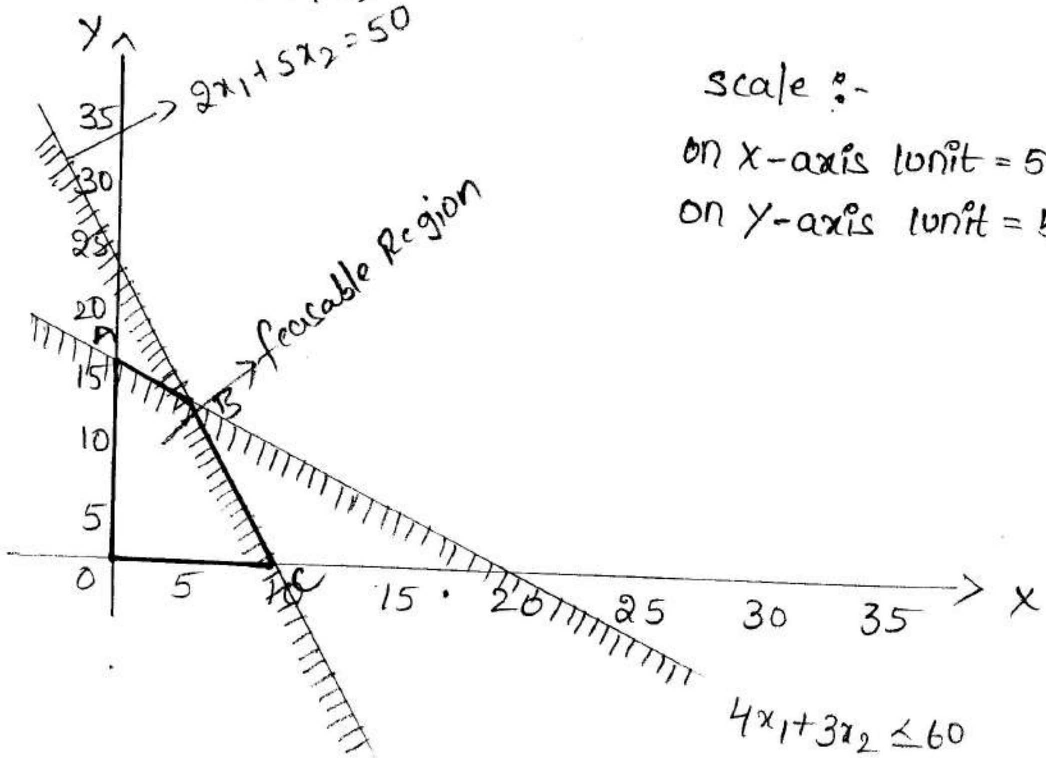
$C(x_1, x_2) = (0, 20)$

$4x_1 + 3(0) = 60$

$4x_1 = 60$

$x_1 = 15$

$D(x_1, x_2) = (15, 0)$



scale :-

on X-axis 1unit = 5cm

on Y-axis 1unit = 5cm

The point B intersect the line

$2x_1 + 5x_2 < 50$ & $4x_1 + 3x_2 < 60$

$2x_1+5x_2=50 \longrightarrow (1)$

$4x_1+3x_2=60 \longrightarrow (2)$

$(1)\times 5 \Rightarrow \cancel{4x}+10x_2=100$

$\cancel{4x_1}+3x_2=60$

$13x_2=160$

$x_2=\frac{160}{3}$

$x_2=12.3$

Substitute in equation (2) $x_2=12.5$

$\Rightarrow 2x_1+5(12.3)=50$

$\Rightarrow 2x_1+61.5=50$

$\Rightarrow 2x_1=50-61.5$

$x_1=\frac{11.5}{2}$

$x_1=5.75$

The Point $B=(x_1,x_2)=(5.75,12.5)$

$A(1,0,0)$, $B(5.7,12.5)$, $C(0,15)$

S.NO	cornerpoint	objectivefunction $\min Z=20x_1+30x_2$
1.	O(0,0)	$20(0)+30(0)=0+0=0$
2	A(10,0)	$20(10)+30(0)=200+0=200$
3	B(5.7,12.5)	$20(5.7)+30(12.5)=114+375=489$
4	C(0,15)	$20(0)+30(15)=0+450=450$

At point o(0,0) the optimal solution can be occured

min $z = 0, x_1 = 0$ & $x_2 = 0$

∴ At point B(5.7, 12.5) the optional solution can be occured

min $z = 489$, $x_1 = 5.7$ & $x_2 = 12.5$

Procedure for solving the simplex method to solve the Lpp :-

step-1 :- check whether the objective function of the given Lpp is to be maximised (or) minimised. if it is to be minimised then we convert it in to a problem of maximisation by Max $z = -\min(-z)$

step-2 :- Express the Problem is standard from by introducing slack (or) surplus variable to convert the inequality constraints into equations.

step-3 :- obtain an Initial basic feasible solution (IBFS) to the problem and put in the secound column in the simplex table.

step-4 :- compute the net evaluations $z_j - c_j$ examing the sign of $z_j - c_j$

* If all $z_j - c_j \geq 0$ then the initial basic feasible solution is an optimum basic feasible solution
* If atleast one $z_j - c_j \ngtr 0$ then Proceed to next step as the solution is not optimal.
* If atleast one $z_j - c_j > 0$ then Proceed to next step as the solution is not optimal.

step-5 :- To find the entering value (or) variable i.e key column if there are more than the negative $z_j - c_j$ choose the most negative of them. This gives the entering variables and is indicated by an arrow at the bottom of the column. If there are more than one variable having the same most negative $z_j - c_j$ then any of them can be selected orbitary as the entiring variable.

Step-6 :- To find the leaving variable (or) key row compute minimum ratio = x_B / x_K

Where x_B = Basic variables

x_K = Key column.

Elect the minimum ratio then choose the variable to leave the basis called the key row and the element all the intersection of key row & key column is called key element.

Step-7 :- form a new basis by droping the leaving variable and introducing the entering variable along with the associated value under column. convert the leaving element to unit by dividing the key equation by the key element and all other elements in its column to zero. by using guass element equation. method and the formula

$$\text{New element} = \text{old element} - \left[\frac{\text{Product of key row \& key column elements}}{\text{Key element}} \right]$$

Step-8 :- compute the net evaluation $z_j - c_j$ untill eiethea an optimum solution is obtained there is an indication of unbounded solution.

Note Points :-

* check whether the given objective function is minimised / maximised.
* objective function is maximised no need to convert.
* objective function is minimised we convert to Maximization.

$$\text{Min } z = -(\text{Max } z)$$

* Subjective constraints is $\leq$ we have to introduce the slake variables $= s_1, s_2, s_3 \cdots\cdots s_n$.

ex:- $3x_1 + 5x_2 \leq 10$

$3x_1 + 5x_2 + s_1 = 10$

Basic variables	c_B	c_j / x_B	a_1 x_1	a_2 x_2	----	a_n x_n	0 s_1	0 s_2	---	0 s_n	Min. ratio $= x_B / x_K$
s_1	0	a_1	a_{11}	a_{12}	---	a_{1n}	1,	0	---	0	10
s_2	0	a_2	a_{21}	a_{22}	---	a_{2n}	0	1	---	0	15
⋮	⋮	⋮	⋮	⋮		⋮					⋮
s_n	0	a_n	a_{n1}	a_{n2}	----	a_{nn}	0	0	---	1	1 ← keyrow
	$z_j = c_B \times x_B$ $\Delta_j = z_j - c_j$									Key element	

where x_B = values of the subject to constraints
x_K = key column elements.

* calculate z_j value z_j = sum of the product of c_B and x_B values $(c_B \times x_B)$
* calculate $\Delta_j = z_j - c_j$

where c_j = constrant values of the normalised objective function (or) modified.

* calculate key column

key column = most negative element existed in $z_j - c_j$

→ key column is the entering variable and key row is the living variable. Intersection of both key row & key column elements.

key row = minimum ratio.

min ratio = x_B / x_K

* All the old elements except key row & key column should be replaced with new elements.

$$\text{new element} = \text{old element} - \frac{\text{Product of key row \& key column element}}{\text{key element}}$$

-> Key row must be divided with key element.
-> Key column except key element all elements are zero's in the key column.

Condition :-
If all Δ_j values >0 then we have the optimal solution.

* Solve the LPP

$$Max\ Z = 3x_1+4x_2$$

Subject to $4x_1+2x_2 \le 80$
$2x_1+5x_2 \le 180$
$x_1, x_2 \ge 0.$

Sol:-
Normalized LPP.

$$Max\ Z = 3x_1+4x_2+0S_1+0S_2$$

Subject to constraints $4x_1+2x_2+S_1 = 80$
$2x_1+5x_2+S_2 = 180$
$x_1, x_2, S_1, S_2 \ge 0.$

Basic variables	C_B	C_j / X_B	3 / x_1	4 / x_2	0 / s_1	0 / s_2	min ratio $= x_B/x_K$
s_1	0	80	4	2	1	0	$\frac{80}{2} = 40$
s_2	0	180	2	5	0	1	$\frac{180}{5} = 36$ ← key row
	$Z_j = C_B \times x_B = 0$		0	0 key element	0	0	
	$\Delta_j = Z_j - C_j$		−3	−4	0	0	
s_1	0	8	16/5 → Key element	0	1	−2/5	$8/16 = 8 \times \frac{5}{16} = \frac{40}{16} = 2.5$
	4	36	2/5	1	0	1/5	$\frac{36}{2/5} = 36 \times 5/2 = \frac{180}{2} = 9$
	$Z_j = C_B \times x_B = 144$		8/5	4	0	4/5	
	$\Delta_j = Z_j - C_j$		−7/5 ↑ key column	0	0	4/5	
x_1	3	40/16	1	0	5/16	−1/8	
x_2	4	35	0	1	−1/8	1/4	
	$Z_j = C_B \times x_B = 147.5$ (or) $\frac{2360}{16}$		3	4	7/16	5/8	
	$\Delta_j = Z_j - C_j$		0	0	7/16	5/8	

condition :-

If All $\Delta_j \geq 0$, then the optimal solution existed

here Max $Z = 147.5$ (or) $\frac{2360}{16}$; $x_1 = \frac{40}{16}$, $x_2 = 35$

* solve the LPP

Min $Z = 3x_1 + 2x_2$ subject to $5x_1 + x_2 \le 10$, $2x_1 + 2x_2 \le 12$;

$x_1 + 4x_2 \le 12$, $x_1, x_2 \ge 0$

sol:- Normalised LPP

Min $Z = 3x_1 + 2x_2$

Min $Z = -[\text{Min } Z]$

$= -[3x_1 + 2x_2]$

$= -3x_1 - 2x_2$

Min $Z = -3x_1 - 2x_2 + 0s_1 + 0s_2 + 0s_3$

subject to $5x_1 + x_2 + s_1 = 10$

$2x_1 + 2x_2 + s_2 = 12$

$x_1 + 4x_2 + s_3 = 12$

$x_1, x_2, s_1, s_2, s_3 \ge 0$

Basic Variables	C_B	c_j / x_B	x_1 (−3)	x_2 (−2)	s_1 (0)	s_2 (0)	s_3 (0)	Min ratio x_B/x_K.
s_1	0	10	5	1	1	0	0	
s_2	0	12	2	2	0	1	0	
s_3	0	12	1	4	0	0	1	
	$Z_j = C_B \times x_B = 0$		0	0	0	0	0	
	$\Delta_j = Z_j - c_j$		3	2	0	0	0	

Since All $\Delta_j \geq 0$, then the optimal solution is Existed value here,

$$Min\ z = 0,\ x_1 = 0\ \&\ x_2 = 0$$

* $Max\ z = 9x_1 + 4x_2$

subject to $x_1 + x_2 \leq 100,\ x_2 \leq 60,\ 2x_1 + 3x_2 \leq 450,\ x_1, x_2 \geq 0$.

solve the lpp by using simplex method.

sol:-

$$Max\ z = 9x_1 + 4x_2 + 0s_1 + 0s_2 + 0s_3$$

Subject to $x_1 + x_2 + s_1 = 100$

$x_2 + s_2 = 60$

$2x_1 + 3x_2 + s_3 = 450$

$x_1, x_2, s_1, s_2, s_3 \geq 0$.

Basic variable	C_B	c_j / x_B	9 / x_1	4 / x_2	0 / s_1	0 / s_2	0 / s_3	min ratio $= x_B/x_K$
s_1	0	100	1 key element	1	1	0	0	$\frac{100}{1} = 100$ ← key row
s_2	0	60	0	1	0	1	0	$\frac{60}{0} = \infty$
s_3	0	450	2	3	0	0	1	$\frac{450}{2} = 225$
	$z_j = C_B \times x_B = 0$		0	0	0	0	0	
	$\Delta_j = z_j - c_j$		-9 ↑ Key column.	-4	0	0	0	
x_1	9	100	1	1	1	0	0	
s_2	0	60	0	1	0	1	0	
s_3	0	250	0	1	-2	0	1	
	$z_j = C_B \times x_B = 900$		9	9	9	0	0	
	$\Delta_j = z_j - c_j$		0	5	9	0	0	

∴ since all the $\Delta j \geq 0$ then the optimal solution can be existed here

$$Max\ Z = 900,\ x_1 = 100,\ x_2 = 0$$

* solve the lpp max $Z = 3x_1 + 2x_2$

subject to constraints $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

Sol:-

$$Max\ Z = 3x_1 + 2x_2 + 0s_1 + 0s_2$$

Subject to $x_1 + x_2 + s_1 = 4$

$x_1 - x_2 + s_2 = 2$

$x_1, x_2, s_1, s_2 \geq 0$

Basic variables	C_B	c_j / x_B	3 / x_1	2 / x_2	0 / s_1	0 / s_2	min ratio $= x_B / x_K$
s_1	0	4	1	1	1	0	$4/1 = 4$
s_2	0	2	1	-1	0	1	$2/1 = 2$ ← key row
			Key element				
	$Z_j = C_B \times x_B = 0$		0	0	0	0	
	$\Delta j = Z_j - C_j$		-3 ↑ key column.	-2	0	0	
s_1	0	2	0	2	1	-1	$2/2 = 1$ ← key row
x_1	3	2	1	-1	0	1	$2/-1 = -2$
	$Z_j = C_B \times x_B = 6.$		3	-3	0	3	
	$\Delta j = Z_j - C_j$		0	-5 ↑ keycolumn.	0	3	

x_2	2	1	0	1	$\frac{1}{2}$	$-\frac{1}{2}$
x_1	3	3	1	0	$\frac{1}{2}$	$\frac{1}{2}$
	$Z_j = C_B \times x_B = 11$		3	2	$\frac{5}{2}$	$\frac{1}{2}$
	$\Delta_j = Z_j - C_j$		0	0	$\frac{5}{2}$	$\frac{1}{2}$

Since all $\Delta_j \geq 0$, then the optimal solution can be existed here; $\max z = 11$, $x_1 = 3$, $x_2 = 1$

IMP

* Use simplex method to solve the LPP ~~Max z =~~

$\text{Max } Z = x_1 + x_2 + 3x_3$, subject to $3x_1 + 2x_2 + x_3 \leq 3$, $2x_1 + x_2 + 2x_3 \leq 2$, $x_1, x_2, x_3 \geq 0$.

sol:- $\text{Max } Z = x_1 + x_2 + 3x_3 + 0s_1 + 0s_2$

subject to
$$3x_1 + 2x_2 + x_3 + s_1 = 3$$
$$2x_1 + x_2 + 2x_3 + s_2 = 2$$
$$x_1, x_2, s_1, s_2 \geq 0.$$

Basic variable	C_B	C_j / x_B	1 / x_1	1 / x_2	3 / x_3	0 / s_1	0 / s_2	min ratio $= x_B/x_K$.
s_1	0	3	3	2	1	1	0	$3/1 = 3$
s_2	0	2	2	1	2	0	1	$2/2 = 1$ ← key row.
	$Z_j = C_B \times x_B = 0$		0	0	0	0	0	
	$\Delta_j = Z_j - C_j$		-1	-1	-3 ↳ key column.	0	0.	

↳ key element

S_1	0	2	2	$3/2$	0	1	$-1/2$
x_3	3	1	1	$1/2$	1	0	$1/2$
	$z_j = C_B \times x_B = 3$		3	$3/2$	3	0	$3/2$
	$\Delta_j = z_j - c_j$		2	$1/2$	0	0	$3/2$

Here all $\Delta_j \geq 0$, then the optimal solution can be existed.

Here $\max z = 3$; $x_3 = 1$

Artificial variable technique method :-

The LPP is in which constraints may also have $\geq$ and equal to (=) signs after ensuring that all values are consider in this section.

The artificial variable technique can be existed in two cases.

1) charne's Big-m method
(or)
method of Penalities.

2) Two-phase simplex method.

* Charne's Big-m method :-

The following steps are involved in solving an Lpp using the Big-m method.

1) Express the problem in the standard form

2) Add non-negative artificial variables to the left side of each of the equations corresponding to the constraints of the type ≥ (or) = . However addition of these artificial variables can uses violation of the corresponding constraints.

3) Solve the modified Lpp by simplex method. until any of these 3 cases may arraise.

case-1 :- if no artificial variable ~~is these in the basis at zero level and the optimum~~ appears in the basis and the optimality conditions are satisfied then the current solution is an optimal basic feasible solution.

case-2 :- if atleast one artificial variable is there in the basis at zero level and the optimality conditions is satisfied. Then the current solution is optimal basic feasible solution.

case-3 :- If atleast one artificial variable appears in the basis at positive level and the optimality condition is satisfied level then the current solution is optimal basic feasible solution.

case-4 :- The solution satisfy the constraints but does not optimal the objective function since it contains a very large penality 'M' and is called "pseudo"

* solve the lpp $\max z = 3x_1+2x_2$

subject to $2x_1+x_2 \leq 2$

$3x_1+4x_2 \geq 12$ and

$x_1, x_2 \geq 0$

Sol:- $\text{Max } Z = 3x_1+2x_2+0S_1+0S_2-mA_1$

subject to $2x_1+x_2+S_1 = 2$

$3x_1+4x_2+S_2+A_1 = 12$

$x_1, x_2, S_1, S_2, A_1 \geq 0$

Note:- If artificial variable is the leaving variable (or) key row then we have neglect the artificial variable column.

Basic variable	C_B	c_j / X_B	3 / x_1	2 / x_2	0 / s_1	0 / s_2	$-m$ / A_1	max ratio $= x_B/x_K$.
s_1	0	2	2	1	1	0	0	$2/1 = 2 \leftarrow$ key row
A_1	$-m$	12	3	4	0	-1	1	$\frac{12}{4} = 3$
	$z_j = C_B \times x_B = 12m$.		$-3m$	$-4m$	0	m	$-m$	
	$\Delta_j = z_j - c_j$		$-3m-3$	$-4m-2$	0	m	0	
x_2	2	2	2	1	1	0	0	
A_1	$-m$	4	-5	0	-4	-1	1	
	$z_j = 4-4m$		$4+5m$	2	$2+4m$	m	$-m$	
	$\Delta_j = z_j - c_j$		$1+5m$	0	$2+4m$	m	0.	

Key element

Since all $\Delta_j \geq 0$ and 1 artificial variable in the basis then the optimal condition is satisfied. but there is no feasible solution then the solution is called "pseduo" optimal solution.

* Solve the LPP $\max(z) = 4x_1 + x_2$

Subject to
$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

Sol: $\text{Max } z = -(4x_1 + x_2)$

$\text{Max } z = -4x_1 - x_2 - mA_1 + 0S_1 - mA_2 + 0S_2$ (or)

$-4x_1 - x_2 + 0S_1 + 0S_2 - mA_1 - mA_2$

subject to $3x_1+x_2+A_1=3$

$4x_1+3x_2-S_1+A_2=6$

$x_1+2x_2+S_2=4$ and

$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0.$

Basic variable	C_B	c_j / x_B	-4 / x_1	-1 / x_2	0 / S_1	0 / S_2	$-m$ / A_1	$-m$ / A_2	min ratio $= x_B/x_K$
A_1	$-m$	3	3	1	0	0	1	0	$3/3 = 1 \leftarrow$ key row
A_2	$-m$	6	4	3	-1	0	0	1	$6/4 = 3/2 = 1.5$
S_2	0	4	1	2	0	1	0	0	$4/4 = 1$
	$Z_j = C_B \times x_B = -9m$		$-7m$	$-4m$	m	0	$-m$	$-m$	
	$\Delta_j = Z_j - c_j$		$-7m+4$ (Key column)	$-4m+1$	m	0	0	0	
x_1	-4	1	1	$1/3$	0	0	0	0	$1/(1/3) = 1/6 \times 3/1 = 3$
A_2	$-m$	2	0	$5/3$	-1	0	-	-	$2/(5/3) = 6/5 \geq 1 \leftarrow$ key row
x_2	0	3	0	$5/3$	0	1	-	-	$3/(5/3) = 9/5 = 1.8$
	$Z_j = -4-2m$		-4	$\frac{-4-3m}{3}$	m	0	-	$-m$	
	$\Delta_j = Z_j - c_j$		0	$\frac{-1-5m}{3}$ (Key column)	m	0	-	0	
x_1	-4	$3/5$	1	0	$1/5$	0	-	-	$\frac{3/5}{1/5} = 3/1 = 3$
x_2	-1	$6/5$	0	1	$-3/5$	0	-	-	$\frac{4/5}{1/5} = -2$
S_2	0	1	0	0	1	1	-	-	$5/1 = 1 \leftarrow$ key row
	$Z_j = C_B \times x_B = -18/5$		-4	-1	$-1/5$	0	-	-	
	$\Delta_j = Z_j - c_j$		0	0	$-1/5$ (Key column)	0	-	-	

Key element

Key element

Key element

x_1	-4	$2/5$	1	0	0	$-1/5$	-	-	
x_2	-1	$9/5$	0	1	0	$3/5$	-	-	
s_1	0	1	0	0	1	1	-	-	
	$z_j = C_B \times x_B = -17/5$		-4	-1	0	$1/5$	-	-	
	$\Delta_j = z_j - c_j$		0	0	0	$1/5$	-	-	

Since all $\Delta_j \geq 0$ and no artificial variable in the basis then optimal condition is satisfied.

$\therefore$ The optimal solution

min $z = 17/5$ & $x_1 = 2/5$, $x_2 = 9/5$

Graphical method :-

min $z = 4x_1 + x_2$

s/t $= 3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

Let us consider inequalities in to equations.

$3x_1 + x_2 = 3$

Let $x_1 = 0$ in the equation

$3(0) + x_2 = 5$

$P(x_1, x_2) = (0, 3)$

Let $x_2 = 0$ in the equation.

$3x_1 + 0 = 3$

$\cancel{3}x_1 = \cancel{3}$

$x_1 = 1$

$P_2(x_1, x_2) = (1, 0)$

$4x_1 + 3x_2 = 6$

Let $x_1 = 0$ in the equation

$4(0) + 3x_2 = 6$

$3x_2 = 6$

$x_2 = 2$

$P_3(x_1, x_2) = (0, 2)$

Let $x_2 = 0$ in the equation

$4x_1 + 3(0) = 6$

$4x_1 = 6$

$x_1 = 3/2 = 1.5$

$P_4(x_1, x_2) = (1.5, 0)$

$x_1 + 2x_2 = 4$

Let $x_1 = 0$ in the equation

$0 + 2x_2 = 4$

$2x_2 = 4$

$x_2 = 2$

$\cancel{P_5(x_1, x_2)}$ $P_5(x_1, x_2) = (0, 2)$

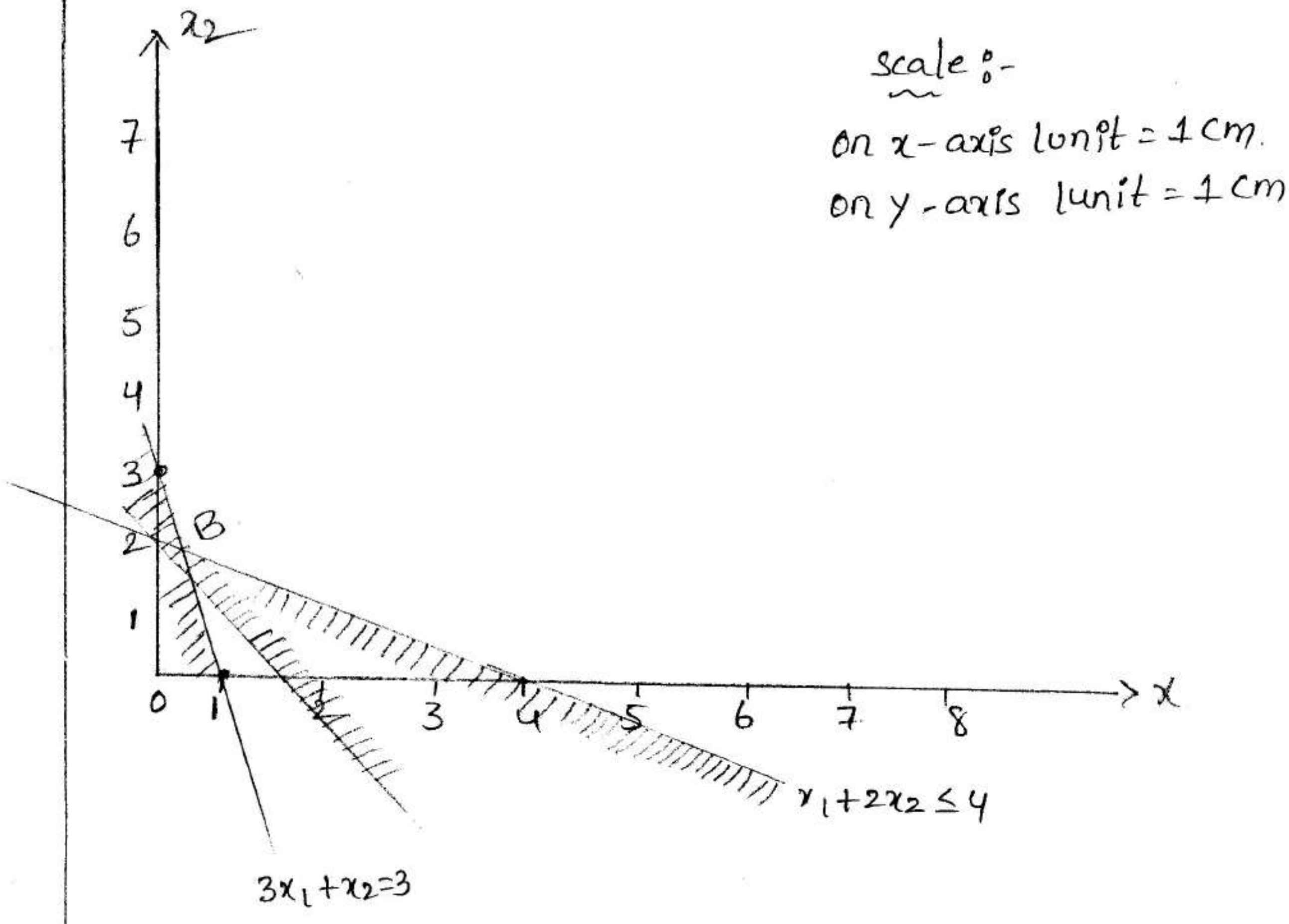
Let $x_2 = 0$ in the equation

$x_1 + 2(0) = 4$

$x_1 = 4$

$P_6(x_1, x_2) = (4, 0)$

plot the points on the graph.



$A(0,2)$

B intersects the line $x_1+2x_2=4$ & $3x_1+x_2=3$

c intersects the line $4x_1+3x_2=6$ & $3x_1+x_2=3$

'B' Intersects the lines $x_1+2x_2=4$ and $3x_1+x_2=3$

$x_1+2x_2=4 \longrightarrow$ (1)

$3x_1+x_2=3 \longrightarrow$ (2)

(1) $\times 3 \Rightarrow \cancel{3x_1}+6x_2=12$

$\cancel{3x_1}+x_2=3$

$- \quad - \quad -$

$5x_2=9$

$x_2=9/5$

substitute $x_2=9/5$ in the equation.

$x_1+2(9/5)=4$

$x_1+18/5=4$

$x_1 = 4 - 18/5$

$x_1 = \frac{20-18}{5}$

$= 2/5$

'c' Intersects the lines

$4x_1 + 3x_2 = 6 \longrightarrow (3)$

$3x_1 + x_2 = 3 \longrightarrow (4)$

$4x_1 + 3x_2 = 6$

$(2) \times 3 \Rightarrow 9x_1 + 3x_2 = 9$

$- \quad - \quad -$

$\cancel{+}\ 5x_1 = \cancel{+}3$

$x_1 = 3/5$

Substitute $x_1 = 3/5$ in the equation (3)

$4(3/5) + 3x_2 = 6$

$12/5 + 3x_2 = 6$

$3x_2 = 6 - 12/5$

$3x_2 = \frac{30-12}{5}$

$x_2 = \frac{18/5}{3} = 18/5$

A(0,2)

B(2/5, 9/5)

C(3/5, 18/5)

$\min z = 4x_1 + x_2$

$(0,2) = 4(0)+2$

$(2/5, 2/5) = 2 = 4(2/5) + 9/5$

$= 8/5 + 9/5$

$= 12/5 = 3.9$

$(3/5, 18/5) = 4(3/5) + 18/5$

$= 12/5 + 18/15 = \frac{36+18}{15} = \frac{44}{15} = 3.6$

* use penalty method solve the LPP min $z = 5x+3y$

s/t $2x+4y \le 12$

$2x+2y = 10$

$5x+2y \ge 10$

sol:- Max $z = -(\min z)$

$= -(5x+3y)$

$= -5x-3y$

$\max z = -5x-3y+0s_1+0s_2-mA_1-mA_2$

s/t $2x+4y+s_1=12$

$2x+2y+A_1=10$

$5x+2y-s_2+A_2=10$ &

$x, y, s_1, s_2, A_1, A_2 \ge 0$

Basic variables	C_B	C_j / X_B	x	y	s_1	s_2	A_1	A_2	Min Ratio $= x_B/x_K$
s_1	0	12	2	4	1	0	0	0	$\frac{12}{2}=6$
A_1	$-m$	10	2	2	0	0	1	0	$10/2 = 5$
A_2	$-M$	10	5 → key element	2	0	-1	0	1	$10/5 = 2$ ← key row
	$Z_j = C_B \times x_B = -20m$		$-7m$	$-4m$	0	M	$-M$	$-M$	
	$\Delta_j = Z_j - C_j$		$-7m+5$	$-4m+3$	0	M	0	0	
s_1	0	8	0	$16/5$	1	$2/5$	0	–	$8/16/5 = \frac{40}{16} = 2.5$ ← key row
A_1	$-M$	6	0	$6/5$	0	$2/5$	1	–	$6/6/5 = \frac{30}{6} = 5$
x	-5	2	1	$2/5$	0	$-1/5$	0	–	$2/2/5 = \frac{10}{2} = 5$

	$Z_j = C_B \times X_B = -6m-10$		-5	$\frac{-6m-10}{5}$	0	$\frac{-2m+5}{5}$	m	$-$	
	$\Delta_j = Z_j - C_j$		0	$\frac{-6m+5}{5}$	0	$\frac{-2m+5}{5}$	0	$-$	
y	-3	$5/2$	0	1	$5/16$	$1/8$	0	$-$	$\frac{5}{2} \times \frac{8}{1} = 20$
A_1	$-m$	3	0	0	$-3/8$	$-1/24$	1	$-$	$3 \times \frac{4}{1} = 12$ ← key row
x	-5	1	1	0	$-1/8$	$-1/4$	0	$-$	$1 \times \frac{4}{7} = 4$
	$Z_j = C_B \times X_B = \frac{-6m-25}{2}$		-5	-3	$\frac{6m-5}{16}$	$\frac{14m+75}{40}$	$-m$	$-$	
	$\Delta_j = Z_j - C_j$		0	0	$\frac{6m-5}{16}$	$\frac{14m+75}{40}$ ↑ key column	0	$-$	
x	-3	1	0	1	$-1/2$	0	$-$	$-$	
S_2	0	12	0	0	$-3/2$	1	$-$	$-$	
x	15	4	1	0	$-1/2$	0	$-$	$-$	
	$Z_j = C_B \times X_B = -23$		-5	-3	1	0	$-$	$-$	
	$\Delta_j = Z_j - C_j$		0	0	1	0	$-$	$-$	

$\therefore$ All $\Delta_j \geq 0$ and no artificial variable exist in the basis then the optimal condition is satisfied.

$\therefore$ The optimal solution is Max $Z^* = 23$.

Min $Z = 23$, $x = 4$ & $y = 1$

Two-phase simplex method :-

The two phase simplex method is an other method to solve the given LPP. The solution is obtained in two-phases.

Phase-1 :-

In this phase we construct an auxilary LPP leading to a final simplex table containing a basic feasable solution to the original Problem.

* Assign a cost -1 to each artificial variable and a cost zero (0) to all other variables and get a new objective function.
* write down the auxillary LPP in which the new objective function is to be maximised, subject to the given set of constraints.
* Solve the auxially LPP by simplex method until either of the following these cases araises.

case (i) :- $Max\ z^* = 0$ and atleast one artificial variable exists in the optimum basis at zero(0) ~~level~~ possible level.

case (ii) :- $Max\ z^* = 0$ and atleast one artificial variable exists in the optimum basis at zero(0) level.

case (iii) :- Max $z^* = 0$ and no artificial variable appears in the optimum basis at Positive level.

Note :-

* In case-1 the given lpp does not posses any feasable solution. then the solution is called pseudo optimal solution.
* In case 2 & 3 exists in the optimum basis then we go to phase-2

Phase - 2 :-

use the optimum basic feasable solution of phase '1' as a starting solution for the original lpp. Assign the actual costs to the variable. in the objective function and a zero (0) cost to every Artificiable variable in the basis at zero level.

Delete the artificial variable column that is eliminated from the basis in phase '1' from the table. Apply simplex method to the modified simplex table obtain at the end of phase 1. till all optimum basic feasable solution is obtained.

* solve the LPP by using two phase simplex method.

$\text{Max } Z = 3x_1 + 3x_2$

$s/t \quad 2x_1 + x_2 \leq 1$

$x_1 + 4x_2 \geq 6$ and

$x_1, x_2 \geq 0.$

Sol:- $\text{Max } Z = 0x_1 + 0x_2 + 0s_1 + 0s_2 - 1A_1$

$s/t \quad 2x_1 + x_2 + s = 1$

$x_1 + 4x_2 - s_2 + A_1 = 6$ &

$x_1, x_2, s_1, s_2 + A_1 \geq 0$

Basic variables	C_B	c_j / x_B	0 / x_1	0 / x_2	0 / s_1	0 / s_2	-1 / A_1	Min ratio $= x_B/x_K$.
s_1	0	1	2	1	1	0	0	$1/1 = 1 \leftarrow$ key row
A_1	-1	6	1	4	0	-1	1	$6/4 = 3/2 = 1.5$
	$Z_j = C_B \times x_B = 6$		-1	-4	0	1	-1	
	$\Delta_j = Z_j - C_j$		-1	-4	0	1	0	
x_2	0	1	2	1	1	0	0	
A_1	-1	2	-7	0	-4	-1	-1	
	$Z_j = C_B \times x_B = -2$		7	0	4	1	-1.	
	$\Delta_j = Z_j - C_j$		7	0	4	1	0	

key element

key column

$\therefore$ All Δ_j value ≥ 0 and $\text{Max } Z^* < 0$ one artificial variable exist in the basis then the optimal condition is satisfied so it doesn't Posses any feasable solution.

$\therefore$ The LPP gives the Pseudo optimal solution.

* Use Two phase simplex method.

$\max Z = 5x_1 - 4x_2 + 3x_3$

$s/t = 2x_1 + x_2 - 6x_3 = 20$

$6x_1 + 5x_2 + 10x_3 \leq 76$

$8x_1 - 3x_2 + 6x_3 \leq 50$ &

$x_1, x_2, x_3 \geq 0$

Sol:

$\text{Max } Z = 0x_1 + 0x_2 + 0x_3 + 0S_1 + 0S_2 - 1A_1$

$s/t \quad 2x_1 + x_2 - 6x_3 + A_1 = 20$

$6x_1 + 5x_2 + 10x_3 + S_1 = 76$

$8x_1 - 3x_2 + 6x_3 + S_2 = 50$ &

$x_1, x_2, x_3, S_1, S_2, A_1 \geq 0$

Basic variables	C_B	C_j / x_B	0 / x_1	0 / x_2	0 / x_3	0 / S_1	0 / S_2	-1 / A_1	Min ratio $= x_B/x_K$
A_1	-1	20	2	1	-6	0	0	1	$20/2 = 10$
S_1	0	76	6	5	10	1	0	0	$\frac{76}{6} = 12.65$
S_2	0	50	8 → key element	-3	6	0	1	0	$\frac{50}{8} = \frac{25}{4} = 6.25$ ↓ key row
	$Z_j = C_B \times x_B = -20$		-2	1	6	0	0	-1	
	$\Delta_j = Z_j - C_j$		-2	-1	6	0	0	0	
A_1	-1	$15/2$	0	$7/4$	$-15/2$	0	$-1/4$	1	$\frac{15}{2} \times \frac{4}{7} = 4.28$ → key row
S_1	0	$77/2$	0	$29/4$	$11/2$	1	$-3/4$	0	$\frac{77}{2} \div \frac{4}{29} = 5.31$
x_1	0	$25/4$	1	$-3/8$	$3/4$	0	$1/8$	0	$\frac{25}{4} = -8/3 = 2.17$

	$Z_j = C_B \times x_B = -15/2$		0	$-7/4$	$15/2$	-1	0	$1/4$	
	$\Delta_j = Z_j - C_j$		0	$-7/4$	$15/2$	0	0	$1/4$	
x_2	0	$30/7$	0	1	$-30/7$	$4/7$	0	$-1/7$	
s_1	0	$\frac{208}{28}$	0	0	$256/7$	$\frac{-29}{7}$	1	$2/7$	
x_1	0	$\frac{220}{28}$	1	0	$-6/7$	$3/14$	0	$1/4$	
	$Z_j = C_B \times x_B = 0$		0	0	0	0	0	0	
	$\Delta_j = Z_j - C_j$		0	0	0	0	0	1	

$\therefore$ All $\Delta_j \geq 0$ with no artificial variable in the basis and $\max z x = 0$ the optimatics condition is satisfies the given Problem as the feasable solution. then we go to Phase-II

Phase-II

considered the final simplex table also consider the original values of the auxiallary auxiallary LPP and eliminate the artificial variable column A_1 in the Phase-II.

Basic Variables	C_B	C_j / x_B	5 / x_1	-4 / x_2	3 / x_3	0 / S_1	0 / S_2	min ratio $= x_B/x_K$
x_2	-4	30/7	0	1	-30/7	0	-1/7	
S_1	0	$\frac{208}{28}$	0	0	256/7	1	2/7	
x_1	5	$\frac{220}{28}$	1	0	-6/7	0	1/14	
	$Z_j = C_B \times x_B = \frac{155}{7}$		5	-4	90/7	0	13/14	
	$\Delta_j = Z_j - C_j$		0	0	69/7	0	13/14	

$\therefore$ All $\Delta_j \geq 0$ and the feasable solution can be existed max $Z = -\frac{155}{7}$, $x_1 = \frac{220}{28}$, $x_2 = \frac{30}{7}$ & $x_3 = 0$

unit – II

Transportation problem.

* Introduction.
* Transportation Model.
* Finding initial basic Feasible solutions.
* Moving towards optimality.
* unbalanced Transportation problems.
* Transportation problems with maximization.
* Degeneracy.

Assignment problem

* Introduction.
* Mathematical Formulation of the problem.
* solution of an Assignment problem.
* Hungarian Algorithm.
* Multiple solution.
* unbalanced Assignment problems.
* Maximization in Assignment model.

Unit - II

Transporation & Assignment

Transporation model :-

-> The origin of transporation model database to 1941 when F.L Hitch cook Present a study entitile "The distribution of a Product from several Source to numerous ~~locatt~~ localities.

-> In 1947 T.C Kopman's Presented a study caused optimum utilization of the transporation problem/ system.

-> These two considerations are mainly responsibility for the development of transporation model which involves shiping sources and a no. of destination.

-> The main objective of transport is to minimise the cost of transporation while meeting the requirements at the destination.

Definations :-

feasible solution :- Any set of non-negative allocations ($x_{ij} > 0$) which satisfies the row and column sum (rim requirement) is called a 'feasible solution'

Basic feasible solution :- A feasible solution is called a "Basic feasible solution" if the number of non-negative allocations is equal to $m+n-1$, where 'm' is the number of rows and n the number of columns in a transportation table.

Non-degenerate basic feasible solution :- Any feasible solution to a transportation Problem containing 'm' origins and n destinations is said to be 'non-degenerate' if it contains $m+n-1$ occupied cells and each allocation is in an independent position.

The allocations are said to be in independent positions, if it is impossible to form a closed path. A path which is formed by allowing horizontal and vertical lines and all the corner cells of which are occupied is called a closed path.

The allocations in the following tables are not in independent positions.

	*	*
	*	*

*		*
*		*

	*	*	
	*		
	*	*	

The allocations in the following tables are in independent positions.

	*	
*	*	*
*		

*	*		
	*		*
		*	*

Degenate basic feasable solution :-

If a basic feasible solution contains less than m+n-1 non-negative allocations, it is said to be 'generate.'

optimal solution :-

optimal solution is a feasible solution (not necessarily basic), which minimizes the total cost.

The solution of a transportation problem can be obtained in two stages, namely initial and optimum solution.

Initial solution can be obtained by using any one of the three methods, viz.,

i, North west corner rule (NWC)

ii, Least cost method (or) Matrix minima method.

iii, vogel's approximation method (VAM).

VAM is preffered over the other two methods, since the initial basic feasible solution obtained by this method is either optimal or very close to the optimal solution.

The cells in the transportation table can be classified as occupied and unoccupied cells. The allocated cells in the transportation table are called occupied cells and the empty ones are called unoccupied cells.

The improved solution of the initial basic feasible solution is called "optimal solution", which is the second stage of solution and can be obtained by MODI (modified Distribution method).

North west corner Rule :-

Step-1 :- starting with the cell at the upper left corner (north west) of the transportation matrix, we allocate as much as possible so that either the capacity of the first row exhausted or the destination requirement of the first column is satisfied i.e $x_{11} = \min(a_1, b_1)$.

Step-2 :- If $b_1 > a_1$, we move down vertically to the second row and make the second allocation of magnitude $x_{22} = \min(a_2 b_1 - x_{11})$ in the cell (2,1).

If $b_1 < a_1$, move right horizontally to the second column and make the second allocation of magnitude $x_{12} = \min(a_1, x_{11} - b_1)$ in the cell (1,2).

If $b_1 = a_1$, there is a tie for the second allocation, we make the second allocations of magnitude

$x_{12} = \min(a_1 - a_1, b_1) = 0$ in cell(1, 2)

(or) $x_{21} = \min(a_2, b_1 - b_1) = 0$ in the cell (2,1)

Step-3 :- Repeat steps 1 and 2 moving down towards the lower right corner of the transportation table untill all the rim requirements are satisfied.

1) obtain the initial basic feasable solution of a Transporation whose cost & requirements are given in the table.

origin	D_1	D_2	D_3	supply
01	2	7	4	5
02	3	3	1	8
03	5	4	7	7
04	1	6	2	14
demand	7	9	18	34

sol:- Here supply = Demand

The given transporation Problem have the initial basic feasable solution.

first allocation :-

origin	D_1	D_2	D_3	supply
01	2|5	7	4	~~5~~
02	3	3	1	8
03	5	4	7	7
04	1	6	2	14
Demand	~~7~~ 2	9	18	34

min (5,7) = 5

Delete 'o_1' row

Second Allocation :-

origin	D_1	D_2	D_3	supply
O_2	3|2	3	1	~~8~~ 6
O_3	5	4	7	7
O_4	1	6	2	14
demand	~~2~~ 0	9	18	34

min(2,8) = 2

Delete 'D_1' column.

Third Allocation :-

origin	D_2	D_3	supply
O_2	3|6	1	~~6~~ 0
O_3	4	7	7
O_4	6	2	14
demand	~~9~~ 3	18	27

min(6,9) = 6

Delete 'O_2' row

Fourth Allocation :-

origin	D_2	D_3	supply
O_3	4|3	7	~~7~~ 4
O_4	6	2	14
demand	~~3~~ 0	18	21

min(3,7) = 3

Delete 'D_2' column

Fifth Allocation :-

origin	D_3	supply
O_3	7] 4	~~4~~ 0
O_4	2	14
demand	14/~~18~~	18

min (4, 18) = 4

Delete 'O_3' row

Sixth Allocation :-

origin	D_3	supply
O_4	2] 14	~~14~~ 0
demand	~~14~~ 0	14

min (14, 14)

Delete 'O_4' row (or) D_3 column.

No. of allocation = $m+n-1 = 4+3-1 = 7-1 = 6$

Total cost $= 2\times5+3\times2 * 3\times6+3\times4+7\times4+2\times14$

=

2) obtain the Initial basic feasable solution the transporation problem by using north west corner method.

	D_1	D_2	D_3	D_4	supply
O1	6	4	1	5	14
O2	8	9	2	7	16
O3	4	3	6	2	5
demand	6	10	15	4	35

Sol: Here supply = demand

The given Transporation Problem have the initial basic feasable solution.

First Allocation :-

	D_1	D_2	D_3	D_4	supply
O_1	6 | 6	4	1	5	~~14~~ 8
O_2	8	9	2	7	16
O_3	4	3	6	2	5
demand	~~6~~ 0	10	15	4	35

$\min(6, 14) = 6$

Delete 'D_1' column.

Second Allocation :-

	D_2	D_3	D_4	supply
O_1	4 | 8	1	5	~~8~~ 0
O_2	9	2	7	16
O_3	3	6	2	5
demand	10	15	4	29

$\min(10, 8) = 8$

Delete 'O_1' row

Third Allocation :-

	D_2	D_3	D_4	Supply
O_2	9|2	2	7	~~16~~ 14
O_3	3	6	2	5
demand	~~20~~	15	4	21

min(2,16) = 2.

Delete 'D_2' column.

fourth Allocation :-

	D_3	D_4	Supply
O_2	2|14	7	~~14~~ 0
O_3	6	2	5
demand	~~15~~ 1	4	21

min (15,14) = 14

Delete 'O_2' row.

Fifth Allocation :-

	D_3	D_4	Supply
O_3	6|1	2	~~5~~ 4
demand	1	4	4

min(1,5) = 1

Delete 'D_3' row.

Sixth Allocation :-

	D_4	Supply
O_3	2|4	4
demand	4	4

min(4,4) = 4

Delete 'D_4' column (or) 'O_3' row.

No. of Allocations = No. of rows & columns = m+n-1

= 3 + 4 - 1

= 7-1

= 6

Total cost => 6×6 + 4×8 + 9×2 + 2×14 + 6×1 + 2×4

= 36+32+18+28+6+8

= 128

Least cost method :-

Step-1 :- Determine the smallest cost in the cell (or) cost matrix in the transporation Problem.

Step-2 :- If $x_{ij} = a_j$ cross the i^{th} row of the transporation table decrease a_i by b_j then go. to step-3

* If $x_{ij} = b_j$ cross the j^{th} column of the transporation table decrease a_i by b_j then go.
* If $x_{ij} = a_i - b_j$ cross either i^{th} row (or) j^{th} column of the transporation table but not both.

Step-3 :- Repeat step-1 & 2 for the resolting reduced transporation table untill all remaining requirements are satisfied. when ever the minimum cost is not unique make an orbitory among the choice of minima.

1)

origin	D_1	D_2	D_3	supply
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	

A) Here the supply = Demand then the given transporation Problem have the IBFS.

First Allocation :-

origin	D_1	D_2	D_3	Supply
O_1	2	7	4	5
O_2	3	3	1 | 8	~~8~~ 0
O_3	5	4	7	7
O_4	1	6	2	14
D	7	9	~~18~~ 10	

min (8,18) = 8

Delete 'O_2' row.

Second allocation :-

origin	D_1	D_2	D_3	supply
O_1	2	7	4	5
O_3	5	4	7	7
O_4	1|7	6	2	$\cancel{14}$ 7
demand	$\cancel{7}$ 0	9	10	19

min(7, 14) = 7

Delete 'D_1' column.

Third Allocation :-

origin	D_2	D_3	supply
O_1	7	4	5
O_3	4	7	7
O_4	6	2|7	$\cancel{7}$ 0
Demand	9	$\cancel{10}$ 3	12

min(7, 10) = 7

Delete O_4' row

fourth Allocation :-

O	D_2	D_3	supply
O_1	7	4|3	$\cancel{5}$ 2
O_3	4	7	7
demand	9	$\cancel{3}$ 0	9

min(3, 5) = 3

Delete 'D_3' column.

Fifth Allocation :-

O	D_2	S
O_1	7	2
O_3	4|7	$\cancel{7}$ 0
demand	$\cancel{9}$ 2	9

$Min(7,9) = 7$

Delete 'O_3' row.

sixth allocation :-

origin	D_2	supply
O	7|2	2
demand	2	2

$min(2,2) = 2$

Delete 'D_2' column.

$\text{Total cost} = 1\times8 + 1\times7 + 2\times7 + 4\times3 + 4\times7 + 7\times2$

$= 83$

$\text{No. of allocations} = m+n-1$

$= 4+3-1$

$= 7-1$

$= 6$

Vogel's approximation method (or) Penality method :-

step-1 :- Find the Penality cost namely the difference b/w smallest and the next smallest costs in each row & column.

Step-2 :- Among the Penalities of founding Step-1 choose the maximum Penality. if these maximum Penality is more then once choose any one (or) orbitary.

Step-3 :- If the selected row (or) column as by step-2 findout the cell having the least cost allocate to this cell as much as Possible depend on the capacity (or) requirement (or) supply & Demand.

Step-4 :- Delete the row (or) column that is fully exhausted again compute the row and column Penality for reduce the transporation table and then go to step-2 repeat the steps untill all the remaining requirements are satisfied.

1) obtain the initial basic feasable solution sorted the given transporation by using vogle's approximation method.

origin	D_1	D_2	D_3	D_4	supply
O_1	11	13	17	14	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
Demand	200	225	275	250	950

Sol:- Here Supply = Demand then the given transporation have the initial basic feasable solution.

First allocation :-

origin	D_1	D_2	D_3	D_4	supply	P.I
O_1	11	13 | 225	17	14	~~250~~ 25	2
O_2	16	18	14	10	300	4
O_3	21	24	13	10	400	3
Demand	200	~~225~~ 0	275	250	950	
P.I	5	5	1	0		

Min (25, 200) = 25

Delete 'D_2' column.

Second Allocation :-

Origin	D_1	D_3	D_4	S	P.I
O_1	11 | 25	17	14	250	3
O_2	16	14	10	300	4
O_3	21	13	10	400	3
demand	~~200~~ 175	275	250	725	
P.II	5	1	0	min (300; 175) = 175	

Delete 'O_1' row.

Third allocation :-

origin	D_1	D_3	D_4	S	P. III
O_2	16) 175	14	10	~~300~~ 125	4
O_3	21	13	10	400	3
demand	~~175~~ 0	275	250	700	
P. II	5	1	0		

Min (300, 175) = 175

Delete 'D_1' column

fourth Allocation :-

origin	D_3	D_4	S	P. IV
O_2	14	10) 25	~~125~~ 0	4
O_3	13	10	400	3
demand	275	~~250~~ 125	525	
P. IV	1	0		

Min (125, 250) = 125 ~~Min (1000, 275) = 275~~

Delete 'O_2' row.

Fifth allocation :-

origin	D_3	D_4	S	P. V
O_3	13) 275	10	~~400~~ 125	3
demand	~~275~~ 0	125	400	
P. V	13	10		

min (400, 275) = 275 ~~min (125, 125) = 0~~

Delete 'D_3' column.

Sixth allocation :-

origin	D_4	S	P.VI
O_3	(10) 125	~~125~~	10
D	~~125~~ 0	125	
P.VI	10		

Min(125, 125) = 125

Delete 'D_4' (or) O_3'

Total no. of allocations = m+n-1
= 3+4-1
= 7-1
= 6

Total cost = 13×225+11×25+16×175+10×125+13×275
+10×125

= 2925+275+2800+1250+3575+1250.

= 12075.

Unbalanced Transporation :-

unbalanced Transporation problem supply not equal to demand there is no initial basic feasable solution existed in the transporation for this purpose we have to introduce dummy row on supply and dummy column in the demand after that the unbalanced transporation can be converted in the balanced transporation then the initial basic feasable solution can be existed.

* If supply < demand we have to add a dummy row with wanted supply.
* If demand < supply we have to add a dummy column with wanted demand.
* Do the Procedure as usually existed in the overall subsactimation method (or) Penality method.

1) obtain the initial basic feasable solution for the transporation problem with the help of over all subsractimation.

origin	D_1	D_2	D_3	D_4	supply
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	70
demand	85	35	50	45	215

Sol: Here supply ≠ Demand that is 195 ≠ 215 we add a dummy row with the help of supply 20 i.e ~~do~~ dummy row = O_4.

After adding the dummy row supply = demand so the given transporation Problem have the initial basic feasable solution.

First allocation :-

origin	D_1	D_2	D_3	D_4	Supply	P_1
O_1	6	1	9	3	70	2
O_2	11	5	2	8	55	3
O_3	10	12	4	7	70	3
O_4	0|20	0	0	0	~~20~~ 0	0
demand	~~85~~ 65	35	50	45	215	
P_1	6	1	2	3		

min(20, 85) = 20

Delete 'O_4' row.

Second allocation :-

origin	D_1	D_2	D_3	D_4	Supply	P_2
O_1	6|65	1	9	3	~~70~~ 5	2
O_2	11	5	2	8	55	3
O_3	10	12	4	7	70	3
demand	~~65~~ 0	35	50	45	115	
P_2	4	4	2	4		

min(70, 65) = 65

Delete 'D_1' column..

~~The~~

Third allocation :-

origin	D_2	D_3	D_4	supply	P_3
O_1	15	9	3	~~8~~ 0	2
O_2	5	2	8	55	3
O_3	12	4	7	70	3
demand	~~35~~ 30	50	45	130	
P_3	4	2	4		

min(5, 35) = 5

Delete 'O_1' row.

Fourth allocation :-

origin	D_2	D_3	D_4	supply	P_4
O_2	5 | 30	2	8	~~55~~ 25	3
O_3	12	4	7	70	3
demand	~~30~~ 0	50	45	125	
P_4	7	2	1		

min(55, 30) = 30

Delete 'D_2' column

Fifth allocation :-

origin	D_3	D_4	supply	P_5
O_2	2 | 25	8	~~25~~ 0	6
O_3	4	7	70	3
demand	~~50~~ 25	45	95	
P_5	2	1		

min(25, 50) = 25

Delete 'O_2' row

Sixth allocation :-

origin	D_3	D_4	supply	P_6
O_3	4	7) 45	~~70~~ 25	3
demand	25	~~45~~ 0	70	
P_6	4	7		

min(70, 45) = 45

Delete 'D_4' column

Seventh allocation :-

origin	D_3	supply	P_7
O_3	4)25	~~25~~ 0	4
demand	~~25~~ 0	25	
P_7	4		

min(25, 25) = 6

Delete 'D_3' (or) O_3 column.

Total no. of allocations $= m+n-1$
$= 4+4-1$
$= 7$

Total cost $= 0\times20 + 6\times65 + 1\times5 + 5\times30 + 2\times25 + 7\times45 + 4\times25$

$= 0 + 390 + 5 + 150 + 50 + 315 + 100$

$= 1010.$

Maximization case in Transporation :-

The objective is to minimize the total Profit for which the Profit matrix is given for this first we have to content the maximization Problem in to minimization by subracting all the elements in the given transportation table the modified minimization Problem will solved for all in the model.

1) solve the following transporation Problem to maximize the Profit.

	A	B	C	D	supply
1	15	51	42	33	23
2	80	42	26	81	44
3	90	40	66	60	33
demand	23	31	16	30	100.

sol:- Here supply = demand, the given transporation have the initial basic feasable solution and also convert the maximisation problem in to minimization by subracting a highest cost in the transporation table.

First allocation :-

	A	B	C	D	supply	P_1
1	75	39	48	57	23	9
2	10	48	64	19	44	1
3	0| 23	50	24	30	~~33~~ 10	24
demand	~~23~~ 0	31	16	30	100.	
P_1	10	9	24	21		

Min(33, 23) = 23.

Delete 'A' column.

Second allocation :-

	B	C	D	supply	P_2
1	39	48	57	23	9
2	48	64	9|30	~~44~~ 14	39
3	50	24	30	10	6
demand	31	16	~~30~~ 0	77	
P_2	9	24	21		

min (44, 30) = 30

delete 'D' column

Third allocation :-.

	B	C	supply	P_3
1	39	48	23	9
2	48	64	14	16
3	50	24|10	~~10~~ 0	26
demand	31	~~16~~ 6	47	
P_3	9	24		

min(10, 16) = 10

delete '3' row.

fourth allocation :-

	B	c	supply	P_4
1	39	48	23	9
2	48|14	64	~~14~~ 0	16 ←
demand	~~31~~ 17	6	37	
P_4	9	16.		

min(14, 31) = 14

delete '2' row.

Fifth allocation :-

	B	c	supply	P_5
1	39	48|6	~~23~~ 17	9
demand	17	~~6~~ 0	23	
P_5	39	48		

min(23, 6) = 6

Delete 'c' column

Sixth allocation :-

	B	supply	P_6
1	39|17	~~17~~ 0	39 ←
demand	~~17~~ 0	17	

Total no. of allocations = m+n-1

min(Total cost) = 0×23 + 9×30 + 24×10 + 48×14 + 48×6 + 39×17

= 0 + 270 + 240 + 672 + 288 + 663 = 2133

$$\text{Max. cost} = 90\times 23 + 81\times 30 + 66\times 10 + 42\times 14 + 42\times 6 + 51\times 17$$
$$= 2070 + 2430 + 660 + 588 + 252 + 867$$
$$= 6867$$

3) There are these factories A, B, C which supply goods to 'y' dealers D_1, D_2, D_3, D_4. The production capacities of these factors are 1000, 700 & 900 units per month respectively. The requirements from the dealers are 900, 800, 500 & 400 units per unit the returns are 8, 7, 7 at these factories. The following table gives unit transporation cost from the factories to the dealers.

	D_1	D_2	D_3	D_4	capacity.
A	2	2	2	4	1000
B	3	5	3	2	700
C	4	3	2	1	900
requirements	900	800	500	400	2600.

Return = Profit + cost

Profit = Return − cost.

Sol:-

	D_1	D_2	D_3	D_4	capacity
A	8-2	8-2	8-2	8-4	1000
B	7-3	7-5	7-3	7-2	700
C	9-4	9-3	9-2	9-1	900
requirements	900	800	500	400	2600.

	D_1	D_2	D_3	D_4	capacity
A	6	6	6	4	1000
B	4	2	4	5	700
C	5	6	7	8	900
Requirements	900	800	500	400	2600

Here we convert Profit matrix in to the Loss matrix with the help of subracting all the cost of the transporation with the highest cost i.e, 8.

	D_1	D_2	D_3	D_4	capacity
A	2	2	2	4	1000
B	4	6	4	3	700
C	3	2	1	0	900
requirements	900	800	500	400	2600

Here capacity = requirements, so the given transporation have the initial basic feasable solution.

First allocation :-

	D_1	D_2	D_3	D_4	capacity	P_1
A	2	2	2	4	1000	0
B	4	6	4	3	700	1
C	3	2	1	0 | 400	~~900~~ 500	1
requirement	900	800	500	~~400~~ 0	2600	
P_1	1	0	1	3		

min(900, 400) = 400

delete 'D_4' column-

Second allocation :-

	D_1	D_2	D_3	capacity	P_2
A	2]900	2	2	~~1000~~ 100	0
B	4	6	4	700	0
C	3	2	1	500	1
Req.	~~900~~ 0	800	500	2200	
P_2	1	0	1		

min (1000, 900) = 900

Delete 'D_1' column.

Third allocation :-

	D_2	D_3	capacity	P_3
A	2	2	100	0
B	6	4]500	~~700~~ 200	2
C	2	1	500	1
Requirement	800	~~500~~ 0	1300	
P_3	0	1		

min(700, 500) = 500

Delete 'D_3' column

Fourth allocation :-

	D_2	capacity	P_4
A	2	100	2
B	6]200	~~200~~ 0	6
C	2	500	2
Req	~~800~~ 0	800	
P_4	0		

min (200, 800) = 200

Delete 'B' row.

Fifth allocation :-

	D_2	capacity	P_5
A	2 | 100	~~100~~ 0	2
C	2	500	
Requirement	~~600~~ 500	600	
P_5	0		

min(100, 600) = 100

Delete 'A' row.

Sixth allocation :-

	D_2	capacity	P_6
C	2 | 500	~~500~~ 0	2
Req.	~~500~~ 0	500	
P_6	2		

min (500, 500) = 500

Delete D_2 (or) C.

Total no. of allocation $= m+n-1$
$= 3+4-1$
$= 7-1$
$= 6$

Total cost $= 0\times400 + 2\times900 + 4\times500 + 6\times200 + 2\times100 + 2\times300$
$= 0 + 1800 + 2000 + 1200 + 200 + 1000$
$= 6200$

$\therefore$ max. Profit $= 8\times400+6\times900+4\times50+2\times200+6\times100$
$+ 6\times500$

$= 3200+5400+2000+400+600+3000$

$= 14600.$

Assignment Problem :-

There are 'n' Jobs to be Perform & 'n' Persons or available for doing this Jobs. Assume that each Person can do each Job at a time.

The Assignment Problem is a special case of the transporation. Problem in which is the objective is to assign a no. of resources to the equal no. of activities at a minimum cost (or) maximum Profit.

Hungarian method :-

Solution of an assignment Problem can be arrived at by using the hungarian method. The steps involved in this method are as follows.

* Prepare a cost matrix if the cost matrix is not a square matrix then add a dummy row co dummy columns based on the requirement.
* Substract the minimum element in each row from all the elements of the respective rows.

* Further the modifing resulting matrix while by subracting the minimum element of each column from all the elements of the respective column. Thus obtain the modify matrix.
* Then drawn the minimum no. of lines i.e, Horizontal and vertical lines to cover all zero's in the resulting matrix. Let the minimum no. of line be 'N'.

Case-1 :- If N=n, where n is the order of matrix then an optimal assignment can be made. so make the assignment to get the required solution.
Case-2 :- If N<n then Proceed to step-5.

Determine the smallest uncovered element in the matrix (elements not covered by 'N' lines).
* Substract the minimum eliment from all uncovered elements and add the same element at the ~~Interaction~~ Intersection of horizontal and vertical lines. thus the second modified matrix is obtained.
* Repeat step3 & 4 untill we get the case 1 of step '4'.
* To make (zero assignment) examing the rows successively untill a row wise exactly single zero is found.

zero(0) this zero is to make the assignment then mark a 'x' (cross) over (or) zero's if lieing in the column of the circled zero, showing that they cannot be considered for further assignment. continue in this manner untill all the zero's have been examined. Repeat the same procedure for column's also.

* Repeat step-6 successively untill one of the following situations araises.

1) If no unmark zero is left then the Process ends.

2) If it is more than one unmarked zero in any column/Row, circle one of the unmarked zero's orbitarly and mark a cross in the cells of remaining zero's in the columns/Rows.

3) Repeat the Process untill no unmarked zero is left in the matrix.

* Thus exactly one marked circle zero in each row & column of the matrix is obtained. The assignment corresponding to those marked circle zero will give the optimal assignment.

Using the following cost matrix Determine

1. optimal Job assaignment
2. The cost of assaignment Job.

Job / Machine	1	2	3	4	5
A	10	3	3	2	8
B	9	7	8	2	7
C	7	5	6	2	4
D	3	5	8	2	4
E	9	10	9	6	10

Sol: Row subraction :- "Least number should be subract"

	1	2	3	4	5
A	(10-2) 8	(3-2) 1	(3-2) 1	(2-2) 0	(8-2) 6
B	7	5	6	0	5
C	5	3	4	0	2
D	1	3	6	0	2
E	3	4	3	0	4

first Modified Matrix :-

	1	2	3	4	5
A	7	0	0	0	4
B	6	4	5	0	3
C	4	2	3	0	0
D	0	2	5	0	0
E	2	3	2	0	2

column substraction :- "Least no. of column subract"

	1	2	3	4	5
A	7	0	0	0	4
B	6	4	5	0	3
C	4	2	3	0	0
D	0	2	5	0	0
E	2	3	2	0	2

Here $N < n$ $(4 < 5)$.

Identify the least element (or) least number i.e 2, subract all uncovered elements with '2' and add the value '2' at the point of intersection araised.
Thus we have to obtain the next modified Matrix.

Second Modified Matrix :-

Jobs / Machine	1	2	3	4	5
A	9	0	0	2	6
B	6	2	3	0	3
C	4	0	1	0	0
D	0	0	3	0	0
E	2	1	0	6	2

Here $N = n$ $(5 = 5)$

The optimal assaignment can be done here.

	1	2	3	4	5
A	9	0	0	2	6
B	6	2	3	0	3
C	4	0	1	0	0
D	0	0	3	0	0
E	2	1	0	6	2

Jobs	Machines	cost
1	D	3
2	A	3
3	E	9
4	B	2
5	C	4

Total cost = 3+3+9+2+4 = 21

2)

Job / Machine	1	2	3	4	5
A	13	8	16	18	19
B	9	15	24	9	12
C	12	9	4	4	4
D	6	12	10	8	13
E	15	17	18	12	20

sol: Row subraction :-

	1	2	3	4	5
A	5	0	8	10	11
B	0	6	15	0	3
C	8	5	0	0	0
D	0	6	4	2	7
E	3	5	6	0	8

we need to convert it in to column subraction

First modified Matrix :-

	1	2	3	4	5
A	5	0	8	10	11
B	0	6	15	0	3
C	8	5	0	0	0
D	0	6	4	2	7
E	3	5	6	0	8

Here N<n (4<5)

Second Modified Matrix :-

	1	2	3	4	5
A	8	0	8	13	11
B	0	3	12	0	0
C	11	5	0	3	0
D	0	3	1	2	4
E	3	2	3	0	5

	1	2	3	4	5
A	8	$\boxed{0}$	8	13	11
B	~~0~~	3	12	~~0~~	$\boxed{0}$
C	11	5	$\boxed{0}$	3	~~0~~
D	$\boxed{0}$	3	1	2	4
E	3	2	3	$\boxed{0}$	5

Job	Machine	cost
1	D	6
2	A	8
3	C	4
4	E	12
5	B	12

Total cost = 6+8+4+12+12 = 42

* Fouor differerent Jobs can be done on four differenee machines the take down time costs are Porohibitively high for change over the matrix below given the cost in Rupees for Poroducing Job 'i' on the machine 'J'.

Job / machines	m_1	m_2	m_3	m_4
J_1	5	7	11	6
J_2	8	5	9	6
J_3	4	7	10	7
J_4	10	4	8	3.

sol: Row subraction :-

	m_1	m_2	m_3	m_4
J_1	0	2	6	1
J_2	3	0	4	1
J_3	0	3	6	3
J_4	7	1	5	0

column subtraction :-

	m_1	m_2	m_3	m_4
J_1	0	2	2	1
J_2	3	0	0	1
J_3	0	3	2	3
J_4	7	1	1	0

first modified matrix :-

	m_1	m_2	m_3	m_4
J_1	0	2	2	1
J_2	3	0	0	1
J_3	0	3	2	3
J_4	7	1	1	0

Second modified Matrix :-

	m_1	m_2	m_3	m_4
A	0	1	1	1
B	4	0	0	2
C	0	2	1	3
D	7	0	0	0

Third modified Matrix :-

	m_1	m_2	m_3	m_4
A	0	0	0	0
B	5	0	0	2
C	0	1	0	2
D	8	0	0	0

Job	machine	cost
m_1	J_1/A	5
m_2	J_2/B	5
m_3	J_3/C	10
m_4	J_4/D	3
		23

unbalanced Assignment Problems :-

1)

Job / machines	A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6

sol:- The given matrix is not a Square matrix. by adding a dummy row i.e, 5th row. After adding a dummy row its should be converted as a square matrix.

Jobs	A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6
5	0	0	0	0	0

Row subraction :-

	A	B	C	D	E
1	2	1	4	0	5
2	0	2	1	4	6
3	3	2	1	0	4
4	2	1	0	3	0
5	0	0	0	0	0

first modified matrix :-

	A	B	C	D	E
1	2	1	4	0	5
2	0	2	1	4	6
3	3	2	1	0	4
4	2	1	0	3	0
5	0	0	0	0	0

Here $N<n$, Identify the least element in the uncovered elements i.e '1' substract one with all uncovered elements and it is added point of Intersection araised.

second modified matrix :-

	A	B	C	D	E
1	2	0	3	0	4
2	0	1	0	4	5
3	3	1	0	0	3
4	3	1	0	4	0
5	1	0	0	1	0

Here N=n the given problem have the optimal assaignment.

	A	B	C	D	E
1	2	~~0~~	3	$\boxed{0}$	4
2	$\boxed{0}$	1	~~0~~	4	5
3	3	1	$\boxed{0}$	~~0~~	3
4	3	1	~~0~~	4	$\boxed{0}$
5	1	$\boxed{0}$	~~0~~	1	~~0~~

Jobs	Machines	cost
1	D	2
2	A	10
3	C	2
4	E	6
5	B	0

Total cost = 2+10+2+6 = 20.

Maximization in Assaignment model :-

The owner of a small machines of has '4' machines available to assign Jobs for the day. '5' Jobs are offered with expected Profit for each machine on each Job are as follows. By using the assaignment method find the assaignment method the Job that should result maximum Profit which Job should be declined.

Jobs	A	B	C	D	E
1	62	78	50	111	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80

sol:- The given matrix is not a sequre matrix by adding row i.e 5th row. after adding a dummy row the assaignment problem can be square Matrix.

Jobs	A	B	C	D	E
1	62	78	50	111	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80
5	0	0	0	0	0

Identify the highest value in the assaignment problem and subract all the elements in matrix with highest value.

Jobs	A	B	C	D	E
1	(111-62) 49	33	61	0	29
2	40	27	50	38	52
3	24	19	0	40	30
4	63	47	24	34	31
5	111	111	111	111	111

Row subraction :-

Jobs	A	B	C	D	E
1	49	33	61	0	29
2	13	0	23	11	25
3	24	19	0	40	30
4	39	23	0	10	7
5	0	0	0	0	0

No need to conveat column subraction.

First modified matrix :-

Jobs	A	B	C	D	E
1	49	33	61	9	29
2	13	0	23	11	25
3	24	19	0	40	30
4	39	23	0	10	7
5	0	0	0	0	0

$N = 4$, $n = 5$

Here $N < n$.

No. of drawn lines is not equal to order of the matrix. Identify the least element in the uncovered element i.e, '7' substract the least uncovered element with all uncovered element and adding to the Point of intersection arraised.

Second modified matrix :-

Jobs	A	B	C	D	E
1	42	26	61	0	22
2	13	0	30	18	25
3	17	12	0	40	23
4	32	16	0	10	0
5	0	0	7	7	0

Here N=n

The order of the matrix = no. of drawn lines.

The optimal assignment is Existed.

Jobs	A	B	C	D	E
1	42	26	61	$\boxed{0}$	22
2	13	$\boxed{0}$	30	18	25
3	17	12	$\boxed{0}$	40	23
4	32	16	$\cancel{0}$	10	$\boxed{0}$
5	$\boxed{0}$	$\cancel{0}$	7	7	$\cancel{0}$

Jobs	machine's	cost
1	D	111
2	B	84
3	C	111
4	E	80
5	A	0

Total cost = 111+84 +111+80 +0

= 386

∴ 5th Job in declined.

Unit – III
Sequencing

* Job sequencing
* Johnsons Algorithm for n jobs and Two Machines.
* n Jobs and Three Machines.

UNIT-III

Sequencing

Introduction :-

In this chapter, we determine an appropriate order (sequence) for a series of Jobs to be done on a finite number of service facilities in some pre-assaigned order, so as to optimize the total cost (time) involved.

Defination :-

Suppose there are n Jobs (1, 2 ---- n), each of which has to be processed one at time at m machines (A, B, C ----). The order of processing each Job through each machine is given. The problem is to find a sequence among $(n!)^m$ number of all possible sequence for processing the Jobs so that the total elapsed time for all the Jobs will be minimum.

Technology and Notations :-

The following are the technologies and notations used in this chapter.

Number of machines :- It means the service facilities through which a Job must pass before it is completed.

Processing order :- It refers to the order in which various machines are required for completing the Job.

Processing time :- It means the time required by each Job on each machine.

Idle time on a machine :-

This is the time for which a machine remains idle during the total elapsed time. The notation x_{ij} is used to denote the idle time of machine j b/w the end of the $(i-1)^{th}$ Job and the start of the i^{th} Job.

Total elapsed time :- This is the time b/w starting the first Job and completing the last Job, which also includes the idle time, if present.

No Passing rule :- It means, Passing is not allowed, i.e maintaining the same order of Jobs over each machine. If each of N-Jobs is to be processed through 2 machines M_1 and M_2 in the order M_1, M_2 then this rule will mean that each Job will go to machine M_1 first and then to M_2. if a Job is finished on M_1, it goes directly to machine M_2 if it is free, otherwise it starts a waiting line or Joines the end of the waiting line, if one already exists. Jobs that from a waiting line are

Processed on Machine M_2 when it becomes free.

Principal Assumptions

i, No machine can process more than one operation at a time.

ii, Each operation once started must be performed till completion.

iii, Each operation must be completed before starting any other operation.

iv, Time intervals for processing are independent of the order in which operations are performed.

v, There is only one machine of each type.

vi, A Job is processed as soon as possible, subject to the ordering requirements.

vii, All Jobs are known and are ready for processing, before the period under consideration designs.

viii, The time required to transfer jobs between machines is negligible.

Type-1 Problems with 'n' Jobs through two machines

The algorithm, which is used to optimize the total elapsed time for processing 'n' Jobs through two machine is called 'Johnson's algorithm' and has the following steps.

consider 'n' jobs (1, 2, 3 ---- n) processing on two machines A and B in the order AB. Processing periods (time) are $A_1 A_2 \cdots A_n$ and $B_1 B_2 \cdots B_n$ as given in the following table.

machine / job	1	2	3 ------ n
A	A_1	A_2	A_3 ----- A_n
B	B_1	B_2	B_3 ---- B_n

The problem is to sequence the jobs as to minimize the total elapsed time.

The solution procedure adopted by Johnson is given below.

step-1 :- select the least processing time occurring in the list $A_1 A_2 \cdots A_n$ and $B_1 B_2 \cdots B_n$. Let this minimum processing time occur for a job K.

step-2 :- If the shortest processing is for machine A, process the K^{th} job first and place it in the begining of the sequence. If it is for machine B, process the K^{th} job last and place it at the end of the sequence.

step-3 :- when there is a tie in selecting the minimum processing time, then there may be three solutions.

i) If the equal minimum values occur only for machine A, select the job with larger processing time in B to be placed first in the job sequence.

ii) If the equal minimum values occur only for machine B, select the job with larger processing time in A to be placed last in the job sequence

iii) If there are equal minimum values, one for each machine, then place the job in machine A first and the one in machine B last.

Step-4 :- Delete the jobs already sequenced. If all the jobs have been sequenced, go to the next step. otherwise, repeat step 1 to 3.

Step-5 :- In this step, determine the overall or total elapsed time and also the idle time on machines A and B as follows.

Total elapsed time = the time b/w starting the first job in the optimal sequence on machine A and completing the last job in the optimal sequence on machine B.

Idle time on A = (Time when the last job in the optimal sequence is completed on machine B) - (Time when the last job in the optimal sequence is completed on machine A).

Idle time on B = when the first job in the optimal sequence starts on machine B + $\sum_{k=2}^{n}$ [time kth job starts on machine B - time (k-1) th job finished on machine B].

Type-II Processing 'n' jobs through three machines A, B, C.

consider 'n' jobs (1, 2 --- n) processing on three machines A, B, C in the order ABC. The optimal sequence can be obtained by converting the problem in to a two-machine problem. From this, we get the optimum sequence using Johnson's algorithm.

The following steps are used to convert the given problem in to a two-machine problem.

step-1 :- find the minimum processing time for the jobs on the first and last machine and the maximum processing time for the second machine.

i.e find $\min_i (A_i, C_i)$ $i = 1, 2 \dots n$ and

$\max_i (B_i)$

step-2 :- check the following inequality.

$$\min_i A_i \geq \max_i B_i$$

(or)

$$\min_i C_i \geq \max_i B_i$$

step-3 :- If none of the inequalities in step 2 are satisfied; this method cannot be applied.

step-4 :- If atleast one of the inequalities in step-2 is satisfied, we define two machines G and H, such that the processing time on G and H are given by

$$G_i = A_i + B_i \quad i = 1, 2 \dots n$$
$$H_i = B_i + C_i \quad i = 1, 2, \dots n.$$

step-5 :- for the converted machines G and H, we obtain the optimum sequence using two-machine algorithm.

Type-III Problems with 'n' Jobs and K machines:

consider n jobs (1, 2 ... n) processing through K machines $M_1, M_2 \dots M_K$ in the same order. The iterative procedure of obtaining an optimal sequence is as follows.

Step-1 :- find min. M_i and min. M_{iK} and Max. of each of M_{i2}, M_{i3} ---- M_{iK1} for $i = 1, 2 ---- n$.

Step-2 check whether

$$\min_i M_{i_1} \geq \max M_{ij}, \text{ for } j = 2, 3 ---- K-1 \text{ (or)}$$

$$\min_i M_{iK} \geq \max_i M_{ij}, \text{ for } j = 2, 3 --- K-1$$

Step-3 :- if the inequalities in step 2 are not satisfied, the method fails, otherwise, go to the next step

Step-4 :- In addition to step 2, if $M_{i2} + M_{i3} + --- M_{iK-1} = c$ where 'c' is positive fixed constant for all $i = 1, 2 - n$ Then determine the optimal sequence for 'n' jobs where the two machines are M_1 and M_K in the order M_1, M_K by using the optimum sequence algorithm.

Step-5 :- If the condition $M_{i2} + M_{i3} + --- M_{iK-1} \neq c$ for all $i = 1, 2 --- n$, we define two machines G and H such that,

$$G_i = M_{i_1} + M_{i_2} + --- + M_{iK-1}$$

$$H_i = M_{i2} + M_{i3} + ---- + M_{iK} \quad i = 1, 2, 3, 4.$$

Determine the optimal sequence of performance of all jobs on G and H using the optimum sequence algorithm for two machines.

Type - IV :- Problems with 2 Jobs through k Machines :-

Consider two ~~machines~~ jobs, each of which is to be processed on k machines $M_1, M_2 --- M_K$ in two different orders. The ordering of each of the two jobs through k machines is known in advance. such ordering may not be the same for both the jobs. The exact or expected processing times on all the given machines are known.

Each machine can perform only one job at a time. The objective is to determine the optimal sequence of processing the jobs so as to minimize total elapsed time.

The optimal sequence in the case can be obtained by making use of the graph.

The procedure is given in the following steps.

step-1 :- First draw a set of axes, where the horizontal axis represents processing time on job 1 and the vertical axis represents processing time on job 2.

step-2 :- Mark the processing time for job 1 and job 2 on the horizontal and vertical lines respectively, according to the given order of machines.

step-3 :- construct various blocks starting from the origin (starting point), by pairing the same machines untill the end point.

step-4 :- Draw the line starting from the origin to the end point by moving horizontally, vertically and diagonally along a line which makes an angle 45^0 with the horizontal line (base). The horizontal segment of this line indicates that the first job is under process will ~~b~~ while second job is idle. similarly, seg the vertical line indicates that the second job is under process while first job is idle. the diagonal segment of the line shows that the jobs are under process simultaneously.

step-5 :- An optimum path is one that minimizes the idle time for both the jobs. Thus, we must choose the path on which diagonal movement is maximum.

step-6 :- the total elapsed time is obtained by adding the idle time for eiether job to the processing time for that job.

Unit - III

Sequencing

Job-sequencing :-

Sequencing gives us an idea of the order in which things can happen in the event

The Problem is to find a sequence among 'n' numbers of all possible sequences for processing the job so that the total clapsed time for all Jobs will be minimum.

Objective :-

The main objective of Job sequencing is to optimise the total cost/ total time involved in the event.

2 machines & n-Jobs.

Johnson's Algorithm for n-Jobs & 2-machines :-

The Algorithm which is used to optimise the total clapsed time for processing n-Jobs through 2-machines is called Johnson's Algorithm

1, * select the least Processing time-occuring in the least $A_1, A_2 \cdots A_n$ & $B_1, B_2 \cdots B_n$.

Let this minimum Processing time occurs for Job-K.

2, * If the shortest processing time is for machine A process the kth Job first and it is placed in the begining of the sequence if it is for machine-B process the kth Job last and it is placed at the end of the sequence.

3, * when there is a tie in a selecting the minimum Processing time then there may be 3 solutions can be existed.

Case-1 :- If the equal minimum values occurs only for machine-A, select the Job with large Processing time in machine-B to be placed first in the Job sequence.

case-2 :- If the equal minimum values occurs only for machine-B select the Job with age Processing time A and to be placed last in the Job sequence.

case-3 :- If there are equal minimum value one for each machine then place the Job in machine-A first and machine-B last.

4, * Delete the Jobs already sequence. If all the Jobs have been sequenced go to the next step otherwise repeat the steps 1, 2 & 3

5, * In this step determine the over all elapsed time (or) total elapsed time & also the idea time on machines A & B as a follows.

Total elapsed Time :-

The time between starting the first Job in the optimal sequence on machine A and completing the last Job in the optimal sequence in machine - B.

Ideal time for machine - A :-

Time when the last Job in the optimal sequence is completed on machine - B. Time when the last Job in the optimal sequence is completed on machine - A.

Ideal time for machine - B :-

When the first Job in the optimal sequence starts on machine - B + $\sum_{K=2}^{n}$ (time K^{th} Job starts on machine - B - Time $(K-1)^{th}$ Job finished on machine - B.

* There are 5 Jobs each of which must go through the 2 machines A&B in the order A, B Processing times are given below. and also determine the sequence of 5 Jobs that will minimise the total elapsed time and also calculate Ideal time for each machine.

Jobs	1	2	3	4	5
machine-A	5	1	9	3	10
Machine-B	2	6	7	8	4

Sol:- Let us findout the minimum Processing time of machines A & B is '1' existed in m-A corresponding Job is '2'

Jobs	1	2	3	4	5
machine-A	5	1	9	3	10
machine-B	2	6	7	8	4

Job sequence A [2 | | | |] B

* The minimum Processing time can be existed in machine 'A' corresponding Job is 2 can be placed in the first of the sequence.

Let us consider minimum Processing time

Jobs	3	4	5
machine-A	9	3	10
machine-B	7	8	4

Job sequence A

2				1

B

* The minimum processing time can be existed in machine 'B' corresponding Job is 1 and it is placed in the last of the sequence.

Jobs	3	4	5
A	9	3	10
B	7	8	4

* The minimum processing time can be existed in machine-A corresponding job is 4 and it will be placed first of the second sequence.

Job sequence

2	4			1

Jobs	3	5
machine-A	9	10
machine-B	7	4

* The minimum Processing

corresponding

Job is 5 and it will be placed in the last of the second sequence (or) next sequence

Job sequencing A | 2 | 4 | 5 | 1 | B.

*

Jobs	3
A	5
B	7

The minimum Processing time is 7 Existed in machine 'B' corresponding Job 4,3 and it will be placed at the last of sequence

Job sequence is

A | 2 | 4 | 3 | 5 | 1 | B

∴ Job sequence is A | 2 | 4 | 3 | 5 | 1 | B

Job sequence	Machine-A		Machine-B		Ideal time		Job sequence
	In	out	In	out	A	B	
2	0	0+1=1	1	1+6=7	0	1	A(A)
4	1	1+3=4	7	7+8=15	0	0	B(I)
3	4	4+9=13	15.	15+7=22	0	0	C(C)
5	13	13+10=23	23	23+4=27	0	1	D(B)
1	23	23+5=28	28	28+2=30	0	1	E(H)
							F(f)
					=0	=3	G(D)
					=30-28+0	=30-30+3	H(E)
					=2	=3	I(G)

∴ Total elapsed time = 30mins

Idle time for machine A is 2 mins

Idle time for machine B is 3 mins

Job sequence 2 → 4 → 3 → 5 → 1

find the sequence that minimise the total elapsed time required to complete the following task on two much. calculate the ideal time for each machines.

Jobs	A	B	C	D	E	F	G	H	I
machine-1	2	5	4	9	6	8	7	5	4
machine-2	6	8	7	4	3	9	3	8	11

A)

Jobs	A	B	C	D	E	F	G	H	I
machine1	2	5	4	9	6	8	7	5	4
machine2	6	8	7	4	3	9	3	8	11

Job sequence.

1 2

A	I	C	B	H	F	D	E	G

Job sequence	machine-1		machine-2		Ideal time	
	In	out	In	out	m-1	m-2
A(A)	0	0+2=2	2	2+6=8	0	2
B(I)	2	2+4=6	8	8+11=19	0	0.
c(C)	6	6+4=10	19	19+7=26	0	0
D(B)	10	10+5=15	26	26+8=34	0	0
E(H)	15	15+5=20	34	34+8=42	0	0
F(f)	20	20+8=28	42	42+9=51	0	0
G(D)	28	28+9=37	51	51+4=55	0	0
H(E)	37	37+6=43	55	55+3=58	0	0
I(G)	43	43+7=50	58	58+3=61	0	0
					61-50+0 =11hrs	61-61+2 =2hrs

Total elapsed time = 61 hrs

Ideal time for machine-1 = 11hrs

Ideal time for machine-2 = 12hrs.

* A company has 6 Jobs A to F all the Jobs have to go on each machine (hrs) is given below. Find the optimal sequence that minimises the total elapsed time and also the ideal time for each machine.

Jobs	A	B	C	D	E	F
machine-1	1	4	6	3	5	2
machine-2	3	6	8	8	1	5

A)

Jobs	A	B	C	D	E	F
machine-1	1	4	6	3	5	2
machine-2	3	6	8	8	1	5

Job sequence.

1 | A | F | D | B | C | E | 2

Job sequence	machine-1 In	machine-1 out	machine-2 In	machine-2 out	Idleal time m-1	Idleal time m-2
A	0	0+1=1	1	1+3=4	0	1
F	1	1+2=3	4	4+5=9	0	0
D	3	2+3=6	9	9+8=17	0	0
B	6	6+4=10	17	17+6=23	0	0
C	10	10+6=16	23	23+8=31	0	0
E	16	16+5=21	31	31+1=32	0	0
					32-21+0 =11hrs	32-32+1 =1hr

∴ Total elapsed Time = 32hrs

Idleal time for machine-1 = 11hrs

Idleal time for machine-2 = 1hrs.

n-Jobs 3-machines :-

* Find the minimum Processing time for Jobs in the first & last machine and the maximum processing time for 2nd machine i.e, minimum (A_i, C_i) where $i = 1, 2, 3 \ldots n$.

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Unit- 3
Sequencing

* n jobs through m machines
* Two jobs and m Machines. problems.

Items	1	2	3	4	5	6	7
cutting	5	7	3	4	6	7	12
sewing	2	6	7	5	9	5	8
Packaging	10	12	11	13	12	10	11

a) Find all order in which these items are Process through this stage and also calculate the minimisation of total time idle time for each. Processing.

b) suppose a third stage of production is introduced (or)

Find the order in which these 7 items are to be Processed and also calculate the total elapsed time and Idle time from Processing.

sol:- Step-1 Let us consider cutting as a sewing as 1 and Packaging as 'c'

$\min(A_i, C_i) = \min(3, 10)$

$\max(B_i) = 9$

Step-2

* $\min(A_i, C_i) \geq \max(B_i)$

$\min(3, 10) \geq \max(9)$

$\min(A_i) \geq \max(B_i)$

$3 \ngeq 9$

* $\min(C_i) \geq \max(B_i)$

$10 \geq 9$

step-3 :- $G = A_i + B_i$

$H = B_i + C_i$

Item	1	2	3	4	5	6	7
G	7	13	10	9	5	12	20
H	12	18	18	18	21	15	19

Job sequence.

G | 1 | 4 | 3 | 6 | 2 | 5 | 7 | H

Job sequence	machine - A		machine - B		machine - C		Ideal time		
	In	out	In	out	In	out	A	B	C
1	0	0+5=5	5	5+2=7	~~7+10=17~~ 7	7+10=17	0	5	7
4	5	5+4=9	7	9+5=14	17	17+13=30	0	2	0
3	9	9+3=12	14	14+7=21	30	30+11=41	0	0	0
6	12	12+7=19 ~~19+7=26~~	21	21+5=26	41	41+10=51	0	0	0
2	19	19+7=26	26	26+6=32	51	51+12=63	0	0	0
5	26	26+6=32	32	32+9=41	63	63+12=75	0	3	0

$\Rightarrow 86-44+0 \Rightarrow 86-52+10 \Rightarrow 86-86+7$

$= 42 \qquad = 44 \qquad = 7$

∴ Total elapsed Time = 86hrs.

Idle time for cutting is 42 hrs.

Idle time for sewing is 44 hrs.

Idle time for Packing is 7hrs.

Job sequence is $1 \to 4 \to 3 \to 6 \to 2 \to 5 \to 7$

n-Jobs through m-machines :-

step-1 :- consider 'n' Jobs 1, 2, 3 ---- n Processing through 'm' machines m_1, m_2 ---- m_K in the same order. The interactive Proced ure of obtaining an optimal sequence is as follows.

Find minimum value minimum (mi_1, mi_K) and $\max(mi_2 \ldots\ldots mi_{K-1})$

step-2 :- check whether $\min(mi_1) \geq \max(mi_j)$ for $J = 2, 3, 4$ ----- $n-1$ (or) $\min(mi_K) \geq \max(mi_j)$

step-3 :- if the inequalities in step '2' are not satisfied. This method fails, otherwise go to next step.

step-4 :- In addition of step-2 are not satisfied. if $mi_2 + mi_3 + \ldots mi_{K-1} = c$ where 'c' Positive fixed constant for all $i = 1, 2, 3 \ldots n$, then determine the optimal solution for the sequence of 'n' Jobs where the 2 machines are m_1 & m_k in the order $m_1 m_k$ by using the optimum sequence algorithm.

step-5 :- If condition $m_{i2}+m_{i3}+\cdots+m_{ik-1} \neq c$ for all $i=1,2,3\cdots 4$. we define the two machines G & H such as $G=m_{i1}+m_{i2}+\cdots+m_{ik-1}$ and

$H=m_{i2}+m_{i3}+\cdots m_{ik}$ where $i=1,2,3\cdots n$.

Determine the optimal sequence and the Perfor mance of all Jobs on G & H using the optimum

* 4 Jobs 1,2,3,4 are to be Processed an each of the 5 machines A, B, C, D & E in the order A, B, C, D, E. find the total minimum elapsed time and also found (or) find out the Ideal time for each machine.

Jobs / machine	1	2	3	4
A	7	6	5	8
B	5	6	4	3
C	2	4	5	3
D	3	5	6	2
E	9	10	8	6

step-1 :- $\min(A_i, E_i) = \min(5, 6)$

$\max(B_i, C_i, D_i) = \max(6, 5, 6)$

step-2 :- $\min(A_i, E_i) \geq \max(B_i, C_i, D_i)$

$\min(5, 6) \geq \max(6, 5, 6)$

$\min(A_i) \geq \max(B_i, C_i, D_i)$

$5 \geq (6, 5, 6)$

$\min(C_i) \geq \max(B_i, C_i, D_i)$

$6 \geq (6, 5, 6)$

step-3 :- Introduce G & H are the 2 newly machines.

$G = A+B+C+D$

$H = B+C+D+E$

Jobs	1	2	3	4
G	17	21	20	16
H	19	25	23	14

Job sequence

G | 1 | 3 | 2 | 4 | H

Job sequence

$1 \to 3 \to 2 \to 4$

Job sequence	A		B		C		D		E	
	In	out	In	out	In	out	In	out	In	out
1	0	0+7=7	7	7+5=12	12	12+2=14	14	14+3=17	17	17+9=26
3	7	7+5=12	12	12+4=16	16	16+5=21	21	21+6=27	27	27+8=35
2	12	12+6=18	18	18+6=24	24	24+4=28	28	28+5=33	33	35+10=45
4	18	18+8=26	26	26+3=29	29	29+4=33	33	33+2=35	45	45+6=51

Idle time

A	B	C	D	E
0	7	12	14	17
0	0	2	4	1
0	2	3	1	0
0	2	1	0	0

m-A = 51-26 = 25

m-B = 51-29 + 11
= 33

m-C = 51-32+18
= 37

m-D = 51-35+19
= 35

m-E = 17+1=18

Total elapsed time = 51

Idle time for A is 25
Idle time for B is 33
Idle time for C is 37
Idle time for D is 35
Idle time for E is 18

①

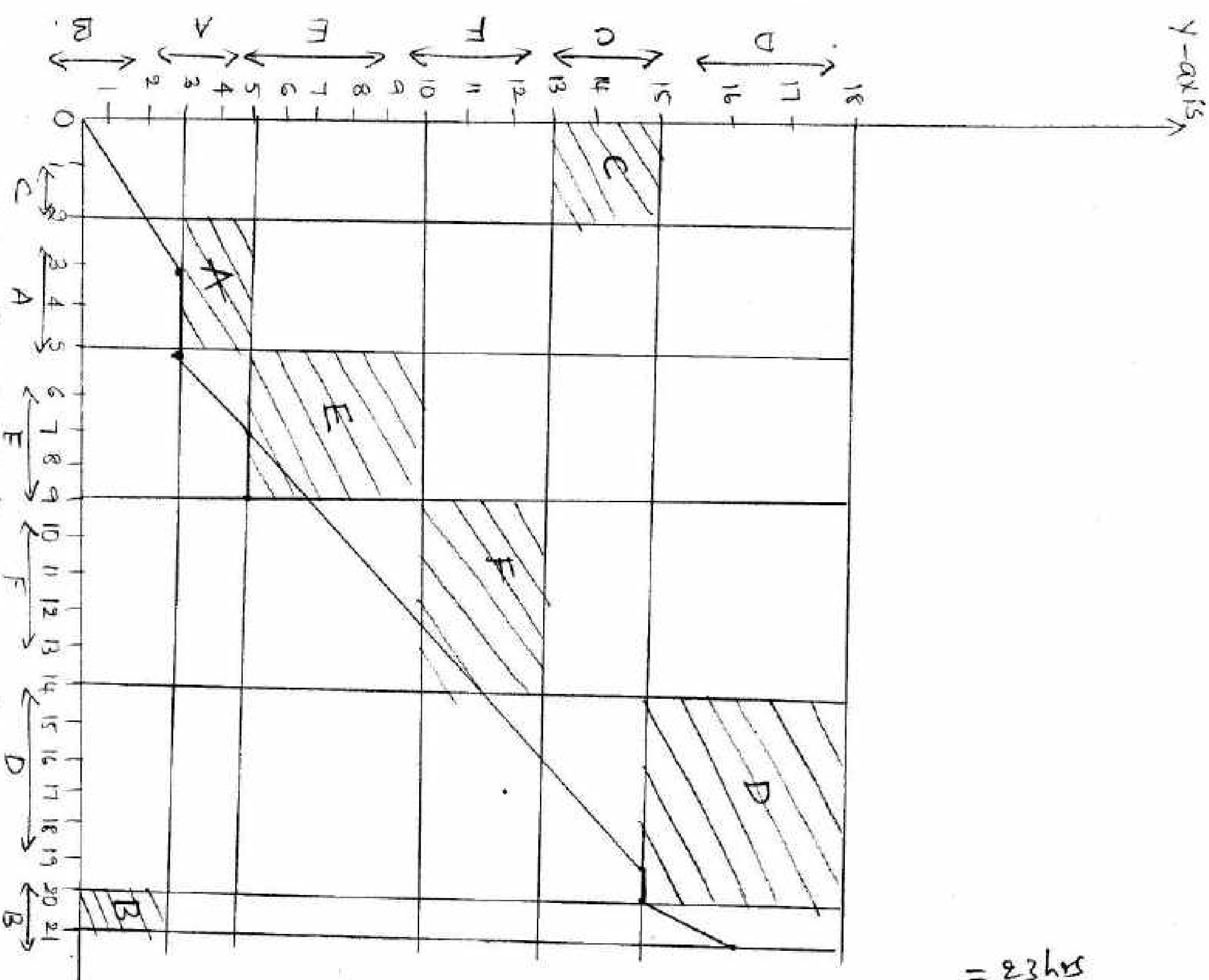
2 Jobs N Machines :—

A Machine sequence : C A E F D B.

Time : 2 3 4 5. 6 1

B Machine sequence : B A E F C D

Time : 3 2 5 3 2 3.



= 23 hrs

Idle time for machine-A on Job-1 is - 21 + 2 ~~= 23 hrs~~.

Idle time for machine-B on Job-2 is - 18 + 2 + 1 + 2 = 23 hrs unit- IV

GAME THEORY.

* concepts.
* Definitions and Terminology.
* Two person zero sum games.
* pure strategy games (with saddle point).
* principal of Dominance.
* Mixed strategy games (game without saddle point).
* significance of game theory in Managerial Application.

UNIT-IV
GAME THEORY

Introduction :-

competition is the watch word of modern life. we say that a competitive solution exists, if two or more individuals make decesions in a situations that involves conflicting interests; and in which the outcome is controlled by the decesion of all the concerned parties. A competative situation is called a 'game'. The term game represents a conflict between two or more parties. A situation is termed a game when it possesses the following properties.

i) The number of competitors is finite

ii) There is a conflict in interests b/w the participants.

iii) Each of the participants have a finite set of possible courses of action.

iv) the rules governing these choices are specified and known to all players. The game begins when each player chooses a single course of action from the list of courses available to him.

v) the outcome of the game is affected by choices made by all the players.

vi) the outcome for all specific set of choices, by all the players, is known in advance and numerically defined.

The outcome of a game consists of a particular set of courses of action undertaken by the competitors. Each outcome determines a set of payments (positive, negative (or) zero), one to each competitor.

Defination :-

The term 'stratagy' is defined as a complete set of plans of action specifing precisely what the player will do under every possible future contigency that might occur doing during the play of the game. i.e stratagy of a player is the decision rule he uses for making a choice, from his list of courses of action. stratagy can be classified as

i) Pure stratagy ii) mixed stratagy.

A stratagy is called pure if one known in advance of the play that it is certain to be adapted, irrespective of the stratagy the other players might choose.

The optimal stratagy mixture for each player may be determined by assaigning to each stratagy, its probability of being choosen. The stratagy so determined is called. mixed stratagy because it is probabilistic combination of the available choices of stratagy. mixed stratagy is denoted by the set,

$S = \{x_1, x_2, \ldots x_n\}$ where x_i is the probability of choosing the courses i such that $x_j \geq 0, j = 1, 2 \ldots n$

Player B

$$\text{Player A}\begin{bmatrix} 1 & 2 & 3 \cdots & j \cdots & n \\ a_{11} & a_{12} & a_{13} \cdots & a_{1j} \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots & a_{2j} \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} \cdots & a_{3j} \cdots & a_{3n} \\ \vdots & & & & \\ m & a_{m2} & a_{m3} \cdots & a_{mj} \cdots & a_{mn} \end{bmatrix}$$

A's Pay off Matrix.

Player B

$$\text{Player A}\begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ m \end{array}\begin{bmatrix} 1 & 2 & 3 \cdots & j \cdots & n \\ -a_{11} & -a_{12} & a_{13} \cdots & -a_{1j} \cdots & -a_{1} \\ -a_{21} & -a_{22} & a_{23} \cdots & -a_{2j} \cdots & a_{2} \\ \vdots & & & & \\ -a_{i1} & -a_{i2} & -a_{i3} \cdots & -a_{ij} \cdots & a_{in} \\ \vdots & & & & \\ -a_{m1} & -a_{m2} & -a_{m3} \cdots & -a_{mj} \cdots & a_{mn} \end{bmatrix}$$

Types of games :-

i) Two person games and n person games :-

In two person games, the players may have many possible choices open to them for each play of the game but the number of players remain only two. Hence, it is called a two-person game. In case of more than two person, the game is generally called n-person game.

ii) Zero-Sum game :- A zero-sum game is one in which the sum of the payment to all the and $x_1 + x_2 + \cdots x_n = 1$. It is evident that a pure strategy is a special case of mixed strategy.

In the case where all about one x_i is zero, a player may be able to choose only 'n' pure strategy, but he has an infinite number of mixed stratagies to choose them.

Pay-off :-

Pay-off is the outcome of playing the game. A pay-off matrix is a table showing the amount received by the player named at the left hand side after all possible plays of the game. The payment is made by the player named at the top of the table.

If a player A has m courses of action and player B has 'n' courses, then a payoff matrix may be constructed using the following steps.

i, Row designations for each matrix are the courses of action available to A.

ii, column designations for each matrix are the courses of action available to B.

iii, with a two perso zero-sum game, the cell entries in B's ~~aff~~ pay off matrix will be the negative of the corresponding entries in A's payoff matrix and the matrices will be as shown below.

competitors is zero, for every possible outcome of the game is in a game if the sum of the points won, equals the sum of the points lost.

iii) Two-person zero-sum game :-

A game with two players, where the gain of one player equals the loss of the other, is known as a two-person zero-sum game. It is also called a rectangular game because their payoff matrix is in the rectangular form. The characteristics of such a game are:

a) only two players participate in the game.
b) Each player has a finite number of stratagies to use.
c) Each specific stratagy results in a payoff.
d) Total payoff to the two players at the end of each play is zero.

The maximin-minimax principle :-

This principle is used for the selection of optimal stratagies by two players, consider two players A and B. A is player who wishes to maximize his gains, while player B wishes to minimize his losses. since A would like to maximize his minimum gains, we obtain for player A, the value called maximum value and the corresponding strategy is called the maximum strategy.

on the other hand, since player B wishes to minimize his losses, a value called the minimax value, which is the minimum of the maximum losses is found. The corresponding strategy is called the minimax strategy. when these two are equal (maximin value = minimax value), the corresponding strategies are called optimal strategies and the game is said to have a saddle point. The value of the game is given by the saddle point.

The selection of maximum and minimax strategy by A and B is based upon the so called maximin-minimax Principle, which guarantees the best of the worst results.

saddle point :-

A saddle point is a position in the payoff matrix, where, the maximum of row minima coincides with the minimum of column maxima. The payoff at the saddle point is called the value of the game.

we shall denote the maximum minimum value by $\underline{v}$, the minimax value of the game $\bar{v}$ and the value of the game by v.

Note :-

i, A game is said to be fair if,

maximin value = minimax value = 0, i.e, if $\bar{v} = \underline{v} = 0$.

ii) A game is said to be strictly determinable if maxmin value = minmax value $\neq 0$. $\underline{r} = r = \overline{r}$

Games without saddle points (mixed stratagies) :-

A game without saddle point can be solved by various solution methods.

2x2 games without saddle point :-

consider a 2x2 two-person zero-sum game without any saddle point, having the payoff matrix for player 'A' as.

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{array}$$

the optimum mixed stratagies,

$$S_A = \begin{bmatrix} A_1 & A_2 \\ p_1 & p_2 \end{bmatrix} \text{ and } S_B \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$$

where,

$$p_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}, \quad p_1 + p_2 = 1 \Rightarrow p_2 = 1 - p_1$$

$$q_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}, \quad q_1 + q_2 = 1 \Rightarrow q_2 = 1 - q_1$$

The value of the game

$$(r) = \frac{a_{11} a_{22} - a_{12} a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

Unit - IV

Game theory

A competition situation is called a game. The term game represents the conflict b/w 2 (or) more parties.

Stratagy :-

The term stratagy is defined as a complete set of plans of an action specifying precisely what the player will do under every possible feature contingency that might occur during the playing of the game.

Pay of Matrix :-

Pay of is the outcome of playing the game a Pay of matrix is the table showing the amount reduced by the player named at left hand side after all possible plays the game the Payment is after all possible plays the game. the Payment in made by the player named at the top of the table.

ex:- $P\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $P\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$, $P\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

Saddle Point :-

A saddle point is position in the pay of matrix. where the maximum of row minima coincides with the minimum of column maxima that is called a ~~...~~ pure strategy (or) saddle point existed.

So, Mathematically it is defined as.

$$\min[\text{column}(\max)] = \max[\text{row}(\text{minima})]$$

(or)

$$\max(\text{row}(\min)] = \min[\text{column}(\max_i)]$$

Note :-

Saddle point is also called max(min) = -min (max) principle.

* Solve the game. whose Pay of matrix is given below

Player-B

$$\text{Player-A}\quad \begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & 1 & 3 & 1 \\ A_2 & 0 & -4 & -3 \\ A_3 & 1 & 5 & 1 \end{array}$$

$$\text{Player-A}\quad \begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \text{Row minima} \\ \hline A_1 & \boxed{1} & 3 & 1 & 1 \\ A_2 & 0 & -4 & -3 & -4 \\ A_3 & \boxed{1} & 5 & 1 & -1 \end{array}$$

column maximum

1 5 1

max(row minima) = min(column maxima)

max(1, -4, -1) = min(1, 5, 1)

1 = 1

saddle point can be existed there with the condition of maximini = minimax ~~Pos~~ principle.

∴ saddle point exists. the value of the game is the saddle point which is '1'. the optimal stratagy in the position of saddle point and is given by A_1, B_1

A_3, B_1

*) Solve the game whose Pay of matrix is given below.

Player 'B'

Player A

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{bmatrix} -2 & 0 & 6 & 5 & 3 \\ 3 & 2 & 1 & 2 & 2 \\ -4 & -3 & 0 & -2 & 6 \\ 5 & 3 & -4 & 2 & 6 \end{bmatrix} \end{array}$$

Player B

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ \text{Column max:} \end{array}\begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{bmatrix} -2 & 0 & 0 & 5 & 3 \\ 3 & 2 & 1 & 2 & 2 \\ -4 & -3 & 0 & -2 & 6 \\ 5 & 3 & -4 & 2 & -6 \end{bmatrix} \\ \begin{array}{ccccc} 5 & 3 & 1 & 5 & 6 \end{array} \end{array}\begin{array}{c} \text{Row minima} \\ -2 \\ 1 \\ -4 \\ -6 \\ \\ \end{array}$$

min (column maximum) = max (Row minimum)

min (5, 3, 1, 5, 6) = max (-2, 1, -4, -6)

1 = 1, Here with Saddle point existed and also the value of the game is 1 and i.s. $A_2 B_3$.

$\Rightarrow$ It is also known as mixed stratagies.

consider a two by two 2 person '0' same game without any saddle point having the pay of matrix for the players A & B as.

$$\begin{array}{cc} & \text{player-B} \\ & \begin{array}{cc} B_1 & B_2 \end{array} \\ \text{player-A} \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{array}$$

The optimal mixed stratagies are

$$S \cdot A = \begin{bmatrix} A_1 & A_2 \\ p_1 & p_2 \end{bmatrix}, \quad S \cdot B \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$$

where $p_1 = \dfrac{a_{22} - a_{21}}{[a_{11} + a_{22}] - [a_{12} + a_{21}]}$

where $p_1 + p_2 = 1$

$p_2 = 1 - p_1$

$q_1 = \dfrac{a_{22} - a_{12}}{[a_{11} + a_{22}] - [a_{12} + a_{21}]}$

$q_1 + q_2 = 1$

$q_2 = 1 - q_1$

The value of the game = $\dfrac{[a_{11} \times a_{22}] - [a_{21} \times a_{12}]}{a_{11}+a_{22} - [a_{12}+a_{21}]}$

1) Solve the following way of matrix. Also determine the optimal stratagies and also the value of the game $A\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}$.

Ans:

$$\begin{array}{c|c} & B \\ A & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array} \quad \begin{array}{c} \text{Row minima} \\ 1 \\ 3 \end{array}$$

column maxima 5 4

max(Row minima) = min(column maxima)

max(1, 3) = min (5, 4)

$3 \neq 4$

∴ the saddle point does not existed.

The mixed stratagies are $SA = \begin{bmatrix} A_1 & A_2 \\ P_1 & P_2 \end{bmatrix}$,

$SB = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$.

$$A \begin{bmatrix} \overset{a_{11}}{5} & \overset{a_{12}}{1} \\ \overset{a_{21}}{3} & \overset{a_{22}}{4} \end{bmatrix} \quad \begin{array}{ll} a_{11} = 5 & a_{12} = 1 \\ a_{21} = 3, & a_{22} = 4 \end{array}$$

where $P_1 = \dfrac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \dfrac{4-3}{(5+4)-(1+3)}$

$= \dfrac{1}{9 \cdot 4} = 1/5$

we know that $p_1+p_2=1$

$$p_2 = 1-p_1$$
$$= 1-1/5$$
$$= \frac{5-1}{5}$$
$$= 4/5$$

$$SA = \begin{bmatrix} A_1 & A_2 \\ 1/5 & 4/5 \end{bmatrix}$$

where

$$q_1 = \frac{a_{22}-a_{12}}{(a_{22}+a_{11})-(a_{12}+a_{21})} = \frac{4-1}{(5+4)-(1+3)} = \frac{3}{9-4}$$
$$= 3/5$$

we know that $\Rightarrow q_1+q_2=1$

$$q_2 = 1-q_1$$
$$= 1-3/5$$
$$= 2/5$$

$$q_2 = \frac{5-3}{5} = 2/5$$

$$S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \\ 3/5 & 2/5 \end{bmatrix}$$

The volume of the game is $\frac{(a_{11}\times a_{22})-(a_{21}\times a_{12})}{(a_{11}+a_{22})-(a_{21}+a_{12})}$

$$= \frac{(5\times 4)-(1\times 3)}{(5+4)-(1+3)} = \frac{20-3}{9-4} = \frac{17}{5}$$

$\therefore$ The mixed stratagies are

$SA = \begin{bmatrix} A_1 & A_2 \\ 1/5 & 4/5 \end{bmatrix}$, $S_B = \begin{bmatrix} B_1 & B_2 \\ 3/5 & 2/5 \end{bmatrix}$ and also the volume of the game is $17/5$.

Q, solve the following game and determine its value and also the optimal stratagies.

$$A \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

A>

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \end{array} \quad \text{Row minima} \quad \begin{array}{c} -4 \\ -4 \end{array}$$

column maxima $4 \quad 4$

max(row minima) = min(column maxima)

$\max(-4, -4) = \min(4, 4)$

$-4 \neq 4$

∴ The saddle point does not existed.

The mixed stratagies are $SA = \begin{bmatrix} A_1 & A_2 \\ P_1 & P_2 \end{bmatrix}$, $SB = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$

$$A \begin{bmatrix} \overset{a_{11}}{4} & \overset{a_{12}}{-4} \\ \overset{a_{21}}{-4} & \overset{a_{22}}{4} \end{bmatrix}$$

$a_{11} = 4$
$a_{12} = -4$
$a_{21} = -4$
$a_{22} = 4$

where $P_1 = \dfrac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$

$= \dfrac{4 - (-4)}{(4+4) - (-4-4)}$

$= \dfrac{4+4}{8 - (-8)}$

$$= \frac{8}{8+8}$$

$$= 8/16 = 1/2$$

$$P_1 = 1/2$$

where $P_1 + P_2 = 1$

$$P_2 = 1 - P_1$$

$$P_2 = 1 - 1/2$$

$$= \frac{2-1}{2} = 1/2$$

where $SA = \begin{bmatrix} A_1 & A_2 \\ P_1 & P_2 \end{bmatrix}$

$$= \begin{bmatrix} A_1 & A_2 \\ 1/2 & 1/2 \end{bmatrix}$$

where $a_1 = \frac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$

$$= \frac{4-(-4)}{(4+4)-(-4-4)}$$

$$= \frac{4+4}{8-(-8)}$$

$$= \frac{8}{8+8} = \frac{8}{10} = 1/2$$

where $q_1 + q_2 = 1$

$$q_2 = 1 - q_1$$

$$= 1 - 1/2$$

$$= \frac{2-1}{2} \Rightarrow 1/2$$

$$= \frac{8}{8+8}$$

$$q_2 = \frac{1}{2}$$

$$S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

value of the game $= \dfrac{(a_{11} \times a_{22}) - (a_{21} \times a_{12})}{(a_{11} + a_{21}) - (a_{21} + a_{12})}$

$$= \frac{(4 \times 4) - (-4 \times -4)}{(4+4) - (-4+(-4))}$$

$$= \frac{16 - (16)}{8 - (-8)}$$

$$= \frac{16-16}{8+8}$$

$$= \frac{0}{16} = 0$$

(3) solve the following game & determine the value and also the optimal stratagies.

$$A \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}.$$

sol:-

$$\begin{array}{cc} \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix} & \begin{array}{c} \text{row min} \\ 2 \\ 1 \end{array} \end{array}$$

column max 4 5

max(row min) = min (column max)

(2 1) = (4 5)

$2 \neq 4$

Saddle point is not existed so the mixed stratagies are

$$S_A \begin{bmatrix} A_1 & A_2 \\ P_1 & P_2 \end{bmatrix}, \quad S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$$

$$A \begin{bmatrix} a_{11} & a_{12} \\ 2 & 5 \\ a_{21} & a_{22} \\ 4 & 1 \end{bmatrix} \quad \begin{array}{l} a_{11} = 2 \\ a_{12} = 5 \\ a_{21} = 4 \\ a_{22} = 1 \end{array}$$

where $P_1 = \dfrac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$

$$= \frac{1-4}{(2+1) - (5+4)}$$

$$= \frac{-3}{3-(9)}$$

$$= \frac{-3}{-6} = 1/2$$

where $P_1 + P_2 = 1$

$$P_2 = 1 - P_1$$

$$= 1 - 1/2$$

$$= 1/2.$$

$P_2 = 1/2$

$$SA = \begin{bmatrix} A_1 & A_2 \\ P_1 & P_2 \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ 1/2 & 1/2 \end{bmatrix}$$

where $q_1 = \dfrac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$

$$= \frac{1-5}{(2+1)-(5+4)}$$

$$= \frac{-4}{3-9}$$

$$= \frac{-4}{6} = 2/3$$

$q_1 = 2/3$

where $q_1 + q_2 = 1$

$$q_2 = 1 - 2/3$$

$$= \frac{3-2}{3} = 1/3$$

$$SB = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix} \begin{bmatrix} B_1 & B_2 \\ 2/3 & 1/3 \end{bmatrix}$$

the value of the game $= \dfrac{(a_{11} \times a_{22}) - (a_{12} \times a_{21})}{(a_{11}+a_{22}) - (a_{21}+a_{12})}$

$$= \frac{(2\times1) - (5\times4)}{(2+1) - (4+5)}$$

$$= \frac{2-20}{3-9} \Rightarrow \frac{18}{6} = 3$$

Principles of dominame Property :-

some times it is observed that one of the pure stratagies of either players is always inferior to atleast one of the remaining the superior stratages said to dominate the interrior once.

In such case of dominance we reduce the size of the war of matrix by deleting those stratagy which are dominated by of hers.

Rules for dominance theory (or) Property :-

1) If all the elements of a row say kth row are ≤ the corresponding elements of any other row. say rth row s then kth row is dominated by rth row.

2) If all the elements of a column said kth columns ≥ to the corresponding elements of any other column. say rth column then the kth column is dominated by rth column.

3) Dominated rows & column may be deleted to reduce the size of the matrix (or) Pay of

the matrix as the optimal stratagy. ~~or~~ will remains uneffected.

4) If some linear combinations of some rows dominates ith row. then ith row will be deleted. Similar agreements follow for columns.

→ use the principle of dominent following the game ~~player~~ player B

$$\text{player A} \begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{bmatrix}$$

sol:- the elements $\begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{bmatrix}$

Since all the elements in the 3^{rd} row $<$ all the elements in the 2^{nd} row.

∴ The 3^{rd} row is dominated by the 2^{nd} row delete the dominated row.

The reduced Pay of matrix is

$$\begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \end{bmatrix}$$

since all the elements in the k^{th} column (or) 3^{rd} column $\geq$ all the elements in the first column.

∴ Third column is dominated by first column.

delete the dominated 3rd column then the reduce pay of matrix is.

$$\begin{bmatrix} 1 & 7 \\ 6 & 2 \end{bmatrix} \begin{array}{c} \text{row min} \\ 1 \\ 2 \end{array}$$

column max 6 7

max(row min) = min (column max)

max (1, 2) = min (6, 7)

max(2) ≠ 6

Saddle point cannot be existed the mixed stratagies are $SA \begin{bmatrix} A_1 & A_2 \\ P_1 & P_2 \end{bmatrix}$, $SB \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$

$$A \begin{bmatrix} a_{11} & a_{12} \\ 1 & 7 \\ a_{21} & a_{22} \\ 6 & 2 \end{bmatrix} \quad \begin{array}{l} a_{11} = 1 \\ a_{12} = 7 \\ a_{21} = 6 \\ a_{22} = 2 \end{array}$$

where $P_1 = \dfrac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$

$$= \frac{2-6}{(1+2)-(7+6)}$$

$$= \frac{-4}{3-13}$$

$$= \frac{4}{10}$$

$$= 2/5.$$

$P_1 = 2/5$

where $P_1 + P_2 = 1$

$P_2 = 1 - P_1$

$= 1 - 2/5$

$= \frac{5-2}{5}$

$= 3/5$

$$SA \begin{bmatrix} A_1 & A_2 \\ P_1 & P_2 \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ 2/5 & 3/5 \end{bmatrix}$$

where $a_1 = \dfrac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$

$= \dfrac{2-7}{(9+2) - (7+6)}$

$= \dfrac{-5}{3-13}$

$= \dfrac{5}{10}$

$q_1 = 1/2$

where $q_1 + q_2 = 1$

$q_2 = 1 - q_1$

$q_2 = 1 - 1/2$

$q_2 = 1/2$

$$\text{So} \begin{bmatrix} B_1 & B_2 \\ 1/2 & 1/2 \end{bmatrix}$$

The volume of the game theory $= \dfrac{(a_{11} \times a_{22}) - (a_{12} \times a_{22})}{(a_{11} + a_{22}) - (a_{21} + a_{12})}$

$$= \frac{1 \times 2 - (7 \times 6)}{1+2 - 7+6}$$

$$= \frac{2-42}{3-13}$$

$$= \frac{-40}{-10}$$

$$= 4$$

$\boxed{\text{game} = 4}$

Graphical Method for mx2 game:—

1. Reduce the size of payoff matrix of player A by Applying the dominance property if it exist.

step 2:— let Y be the probability of selection of Alternative 1 by player B. and '1-y' be the probability of selection of Alternative to by player B.

Derive the Expected game Function of player B. With Respect to each other of the Alternative of player A.

step 3:— Find the value of game, when y=0 and y=1.

step 4:— plot the game Function on a graph by Assuming a suitable scale keep Y on x. axis and the game on Y-axis.

step 5:— Find the lowest Intersection point in the upper boundary of the graph i.e, Minimax point.

step 6:— If the No. of lines, passing through the minimax point is only 2, form a 2x2 payoff matrix Then go to step 8. otherwise go to step 7.

Step 7: Identify any 2 lines with opposite slopes passing through that matrix. Then form a 2x2 Matrix.

Step 8:- solve the 2x2 game using odd and find the strategies for player A and B. and also the value of the game.

2) use the principle of Dominance.

$$\begin{array}{cc} & \text{player B} \\ \text{player A} & \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix} \end{array}$$

Sol:-

$$\begin{array}{ccc} & \text{player B} & \text{Row min} \\ \text{player A} & \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix} & \begin{array}{c} -2 \\ -1 \\ 2 \end{array} \\ \text{col max} & \begin{array}{ccc} 3 & 4 & 6 \end{array} & \end{array}$$

min (column max) = max (Row minima).

min $(3, 4, 6)$ = max $(-2, -1, 2)$

$3 \neq 2$.

Column Dominance:-

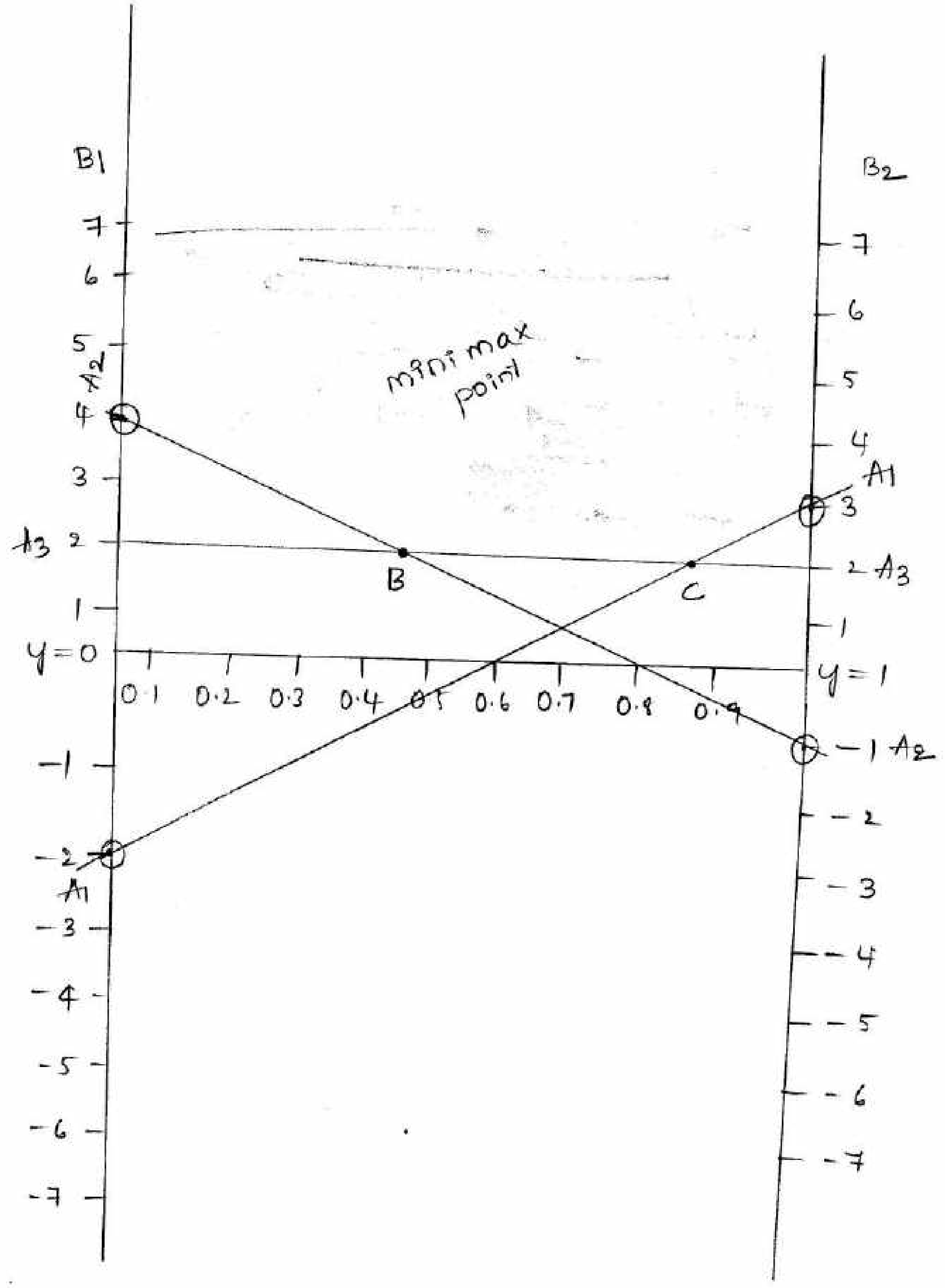
$$\begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \\ 2 & 2 \end{bmatrix}$$

player B

$$\begin{array}{ll|l|l} & \begin{array}{cc} 1 & 2 \end{array} & \text{Row min} & \\ \text{player A} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 3 & -2 \\ -1 & 4 \\ 2 & 2 \end{bmatrix} & \begin{array}{c} -2 \\ -1 \\ 2 \end{array} & \boxed{\begin{array}{c} 1 \\ 3 \\ 4 \end{array}} \\ \text{col max} & \begin{array}{cc} 3 & 4 \end{array} & & \\ & \boxed{\begin{array}{cc} 4 & 4 \end{array}} & & \end{array}$$

A's Alternatives.	gain Function of player B	gain values	
		$y=0$	$y=1$
1	$3y-2(1-y) = 3y-2+2y$ $= 5y-2$	-2	3.
2	$-1y+4(1-y) = -y+4-4y$ $= 4-5y$	4	-1
3	$2y+2(1-y) = 2y+2-2y$ $= 2$	2	2


B1
B2
7
6
5
4
3
2
1
y=0
-1
-2
-3
-4
-5
-6
-7
A2
A3
A1
mini max
point
B
C
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y=1
A1
A3
A2

The No. of passing lines through the Intersection point c is 2. that is A_1 & A_3.
We get the Reduced payoff matrix.

player B.

$$\text{player A}\quad \begin{array}{c} \\ 1 \\ 3 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} \end{array}$$

$a_{11} = 3,\ a_{12} = -2,\ a_{21} = 2,\ a_{22} = -2$

The Mixed strategies A as SA, B as S_B.

$$S_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ P_1 & 0 & P_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_1 = \frac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} + a_{21}}$$

$$P_1 = \frac{2-2}{2+3-(-2+2)}$$

$$P_1 = \frac{0}{+5-0}$$

$$P_1 = 0.$$

$$P_2 = 1 - P_1$$

$$P_2 = 1 - 0$$

$$P_2 = 1.$$

$$S_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ q_1 & q_2 & 0 \\ 4/5 & 1/5 & 0 \end{bmatrix}$$

$$q_1 = \frac{a_{22} - a_{12}}{a_{22} + a_{11} - a_{12} + a_{21}}$$

$$q_1 = \frac{2 - (-2)}{2 + 3 - (-2 + 2)}$$

$$q_1 = \frac{2 + 2}{5 - 0}$$

$$q_1 = 4/5$$

$$q_2 = 1 - q_1$$

$$q_2 = \frac{5 - 4}{5}$$

$$q_2 = 1/5.$$

Value of the game,

$$= \frac{a_{11} \times a_{22} - a_{12} \times a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$= \frac{2 \times 3 - (-2 \times 2)}{2 + 3 - (-2 + 2)}.$$

(4)

$$= \frac{6+4}{5-0}$$

$$= \frac{10}{5}$$

$$= \underline{\underline{2}}$$

* Graphical Method for 2xn game:—

Step 1:– Reduce the size of payoff matrix of player A by Applying the dominance property it is exists.

step 2:— let 'x' be the probability of selection of Alternative one by the player A and '1−x' be the probability of selection of Alternative to by player A.

Derive the Expected game Function of player A with Respect to each of Alternative player B.

step 3:— Find the value of the game where $x=0$ and $x=1$.

step 4:– plot the game Functions on a graph by Assuming a suitable scales keep x on x axis and the game on y-axis.

step 5:— Find the Highest Intersection point in the lower boundary of the graph. i.e, max min point.

Step 6:– If the No. of lines passing through the maxmini point is only 2, form a 2x2 payoff matrix. Then go to step 8 otherwise go to step 7.

Step 7:– Identify any 2 lines with opposite slopes passing through that matrix. Then form a 2x2 matrix.

Step 8:– solve the 2x2 game using oddments and Find the strategies for player A and B and also the Value of the game.

(2) consider the payoff matrix of player A and solve it optimally by using graphical Method.

player B

$$\begin{array}{cc} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \text{player A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} 6 & -7 \\ 1 & 3 \\ 3 & 1 \\ 5 & -1 \end{bmatrix} \end{array}$$

Sol:-

player B.

$$\begin{array}{ccc} & \begin{array}{cc} B_1 & B_2 \end{array} & \text{Row min} \\ \text{player A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} 6 & -7 \\ 1 & 3 \\ 3 & 1 \\ 5 & -1 \end{bmatrix} & \begin{array}{c} -7 \\ 1 \\ 1 \\ -1 \end{array} \\ \text{col max} & \begin{array}{cc} 6 & 3 \end{array} & \end{array}$$

$$\max(\text{Row min}) = \min(\text{col max})$$

$$\max(-7, 1, 1, -1) = \min(6, 3)$$

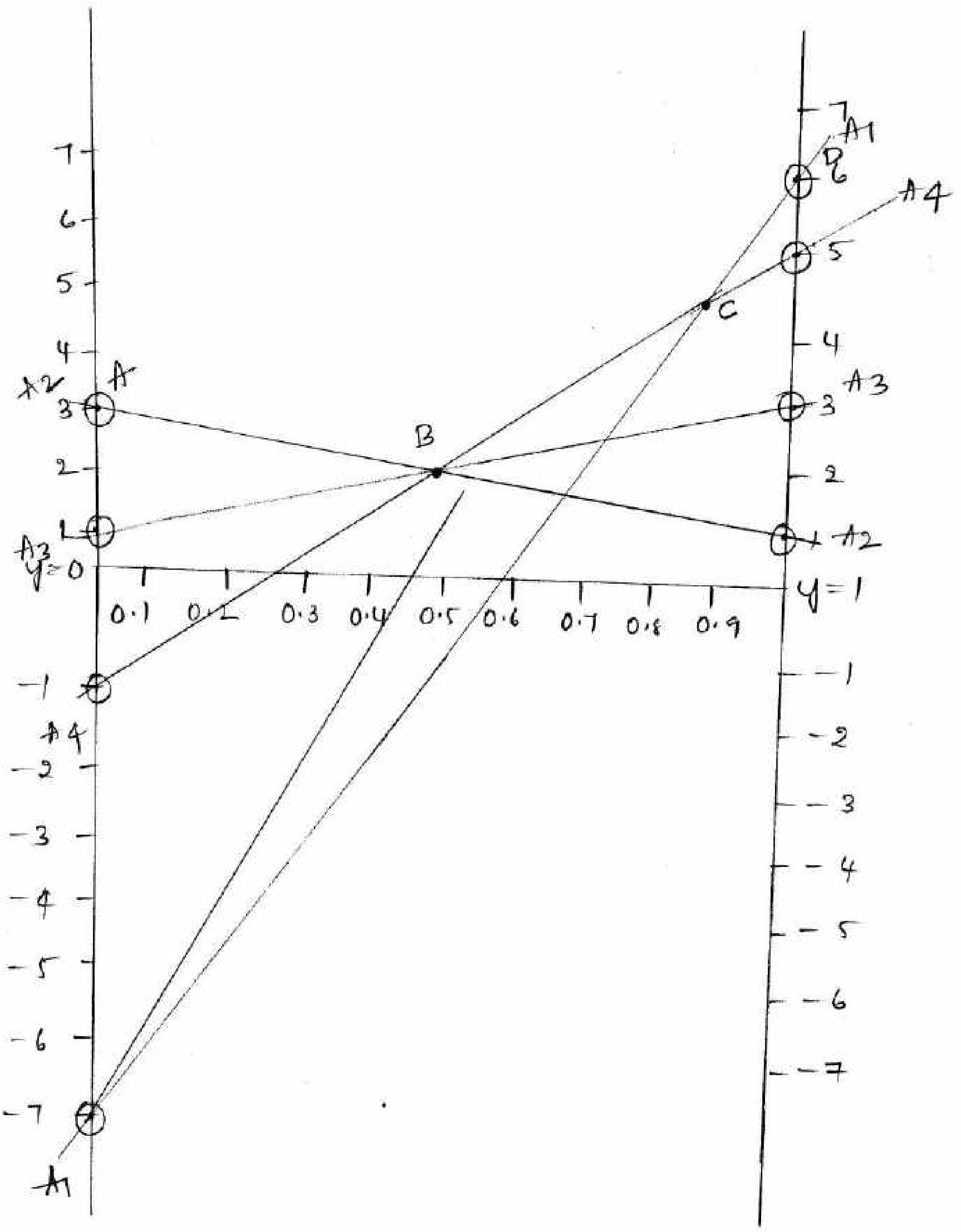
$$1 \neq 3.$$

The given Matrix does not having saddle point.

$$\begin{array}{cc} & \text{player B} \\ \text{player A} & \begin{bmatrix} 6 & -7 \\ 1 & 3 \\ 3 & 1 \\ 5 & -1 \end{bmatrix} \end{array}$$

A's Alter-natives	B's Expected payoff Function.	B's Expected gain	
		$y=0$	$y=1$.
1	$6y-7(1-y) = 6y-7+7y$ $= 13y-7$.	-7	6
2.	$y+3(y-1) = y+3-3y$ $= 2y-3$.	3	1
3	$3y+1(1-y) = 3y+1-y$ $= 2y+1$	1	3
4	$5y-1(1-y) = 5y-1+y$ $= 6y-1$	-1	5.

Plot the Expected B's gain in the graph.



A
B
C
D
A1
A2
A3
A4
y=0
y=1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Reduced pay off matrix, B_1, B_2
A_3, A_4

$$\begin{bmatrix} 1 & 3 \\ 5 & -1 \end{bmatrix}$$

The Mixed strategies of player A is,

$$S_A = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ 0 & 0 & P_1 & P_2 \end{bmatrix} \quad \text{and}$$

$$S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$$

$a_{11} = 3, \; a_{12} = 1, \; a_{21} = 5, \; a_{22} = -1$

$$S_A = P_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$P_1 = \frac{-1 - 5}{3 + (-1) - (1 + 5)}$$

$$P_1 = \frac{-6}{2 - 6}.$$

$$P_1 = \frac{+6}{+4}$$

$$P_1 = \frac{3}{2}$$

$$P_2 = 1 - P_1$$

$$P_2 = 1 - 3/2$$

$$P_2 = \frac{2-3}{2}$$

$$P_2 = 1/2$$

$$S_A = \begin{bmatrix} \cancel{A_1} & \cancel{A_2} & \cancel{A_3} & \cancel{A_4} \\ 0 & 0 & P_1 & P_2 \\ 0 & 0 & 3/2 & 1/2 \end{bmatrix}$$

$$S_B = q_1 = \frac{-1-1}{(-1+3)-(1+5)}$$

$$q_1 = \frac{-2}{2-6}$$

$$q_1 = \frac{+2}{+4}$$

$$q_1 = 1/2$$

$$q_2 = 1 - q_1$$

$$q_2 = 1 - 1/2$$

$$q_2 = \frac{2-1}{2}$$

$$q_2 = 1/2.$$

$$S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \\ 1/2 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 5 & -1 \end{bmatrix}$$

$a_{11} = 1,\ a_{12} = 3,\ a_{21} = 5,\ a_{22} = -1$

$$p_1 = \frac{-1-5}{1+(-1)-(3+5)}$$

$$p_1 = \frac{-6}{0-8}$$

$$p_1 = 3/4$$

$$p_2 = 1/4$$

$q_1 = 1/2,\quad q_2 = 1/2.$

$\therefore$ The value of the game,

$$\frac{a_{11} \times a_{22} - a_{12} \times a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$= \frac{1 \times (-1) - 3 \times 5}{1+(-1) - (3+5)}$$

$$= \frac{-1-15}{0-8}$$

$$= \frac{\cancel{-}16}{\cancel{-}8}$$

$$= \underline{\underline{2}}$$

unit–V

PROJECT MANAGEMENT.

* Rules for drawing the Network diagram.
* Application of CPM. and PERT Techniques in project planning and control.

UNIT - 5
PROJECT MANAGEMENT

Introduction :-

Network scheduling is a technique used for planning and scheduling large projects in the fields of construction, maintenance, fabrication and purchasing of computer system etc. It is a method of minimizing the trouble spots such as production, delays and interruptions, by determining critical factors and coordinating various parts of the overall job.

There are two basic planning and control techniques that utilize a network to complete a predetermined project or schedule. These are program evaluation review technique (PERT) and critical path method (CPM).

A project is defined as a combination of interrelated activities, all of which must be executed in a certain order for its completion.

The work involved in a project can be divided in to three phrases, corresponding to the management functions of planning, scheduling and controlling.

planning :- This phase involves setting the objectives of the project as well as the assumptions to be made, it also involves the listing of tasks or jobs that must be performed in order to complete a project under consideration. In this phase, in addition to the estimates of costs and duration of the various activities, the manpower, machines and materials required for the project are also determined.

scheduling :- this consists of laying the activities according to their order of precedence and determining the following.

i, The start and finish times for each activity.

ii, The critical path on which the activities require special attention.

iii, The slack and float for the non-critical paths.

controlling :- This phase is exercised after the planning and scheduling. It involves the following:

i, Making periodical progress reports.

ii, Reviewing the progress

iii, Analyzing the status of the project.

iv, Making management decesions regarding updating, crashing and resources allocation etc.

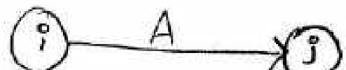
Basic terms :-

To understand the network techniques, one should be familar with a few basic terms of which both CPM and PERT are special applications.

Network :- It is the graphic representation of logically and sequencially connected arrows and nodes, representing activities and events in a Project. Networks are also called arrow diagrams.

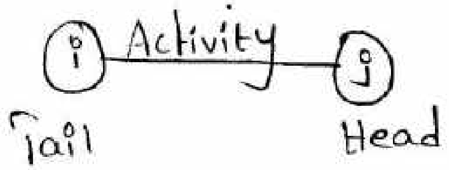
Activity :- An activity represents some action and is a time consuming effort necessary to complete a particular part of the overall project. Thus, each and every activity has a point of time where it begins and a point where it ends.

It is represented in the network by an arrow

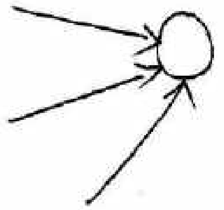


Here 'A' is called the activity.

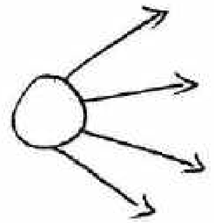
Event :- The begining and end points of an activity are called events or nodes. Event is a point in time and does not consume any resources. It is represented by a numbered circle. The head event called the jth event always has a number higher than the tail event, which is also called the ith event.



Merge and Burst events :- It is not necessary for an event to be the ending event of only one activity as it can be the ending event of two (or) more activities, such an event is defined as a merge event.



If the Event happens to be the begining event of Two or more activities, it is defined as a burst event.



Preceding, suceeding and concurrent activities :- Activities that must be accomplished before a given event can occur, are termed as Preceding activities.

Activities that cannot be accomplished conco concurrently accomplished untill an event has occured, are termed as succeeding activities.

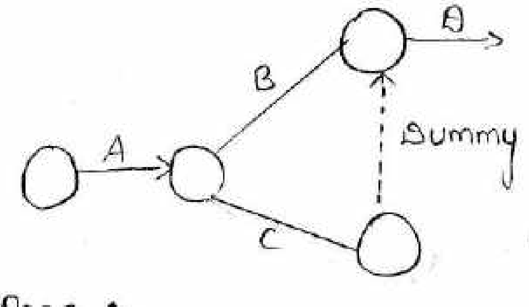
Activities that can be accomplished concurrently, are known as concurrent activities.

The classification is relative, which means that one activity can be Preceding to a certain event, and the same activity can be succeeding to some other event or it may be a concurrent activity with one (or) more activities.

Dummy activity :- certain activities, which neither consume time nor resources but are used simply to represent a connection or a link b/w the events are known as dummies. It is shown in the network by a dotted line. The Purpose of introducing dummy activity is.

i, To maintain uniqueness in the numbering system, as every activity may have a distinct set of events by which the activity can be identified.

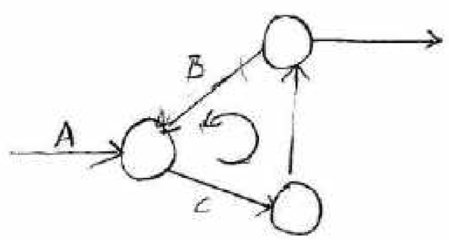
ii, To maintain a Proper logic in the network.



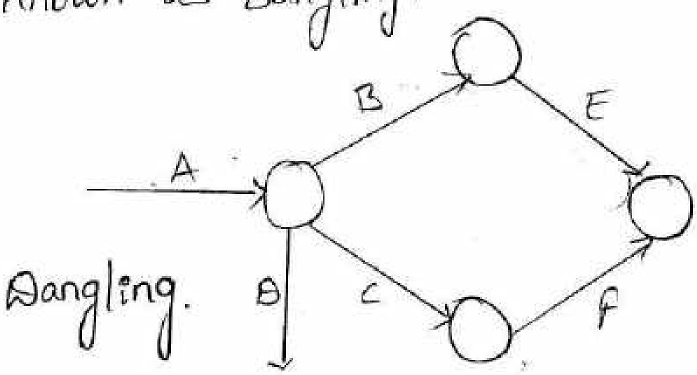
Common ERRors :-

following are the three common errors in a network constructions.

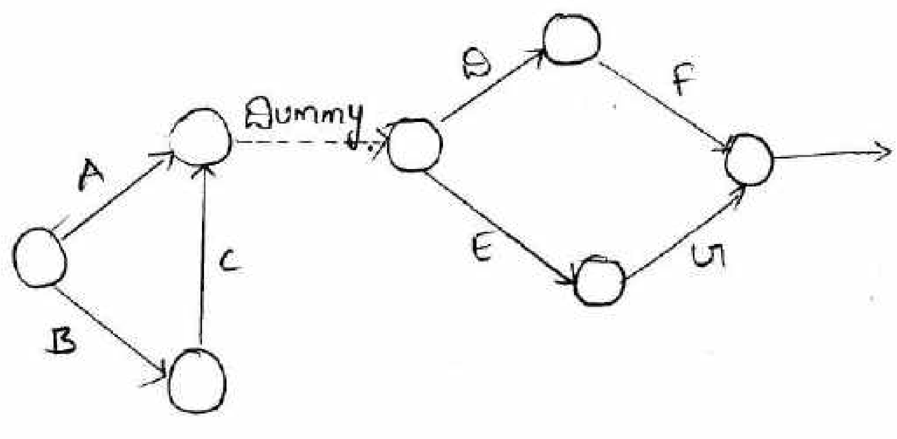
Looping (cycling) :- In a network diagram, a looping error is also known as cycling error. Drawing an ~~ending~~ endless loop in a network is known as cycling error. Drawing an endless of looping. A loop can be formed if an activity is represented as going back in time.



Dangling :- To Disconnect an activity before the completion of all the activities in a network diagram, is known as dangling.



Redundancy :- If a dummy activity is the only activity emanating from an event and can be eliminated, it is known as redundancy.



critical Path method :-

The critical path method is step by step Procedure for scheduling the activities in project is an important tool related to, effective project management.

The iterative procedure of determining the critical Path is as follows.

step-1 :- List all the Jobs and then drawn an. as row diagram. each Job is indicated by an arrow with the direction of the arrow showing the sequence of Jobs.

step-2 :- Indicate the normal time (Tij) for each activity (i,j) above the arrow which is deterministic.

step-3 :- calculate the earliest start time and the earliest finished time (ei) for each event i, in the [v] and also calculate the latest finished time and latest start time. from this we calculate the latest time (LT) for each event and put in the [ ][v]

step-4 :- Tabulate the various times namely normal time, earliest time and latest time anounce diagram.

step-5 :- Determine the total flow for each activity by taking the different b/w earliest starting time and the latest starting time. Mathematically it is denoted by total float ES – LS.

step-6 :- Identify the critical values and connect them with the begining and ending events in the. network diagram by double line arrows. This gives the critical path.

step-7 :- calculate the total Path duration.

critical Path :- The sequence of critical activities in a network which determines the duration of a Project is called the critical Path.

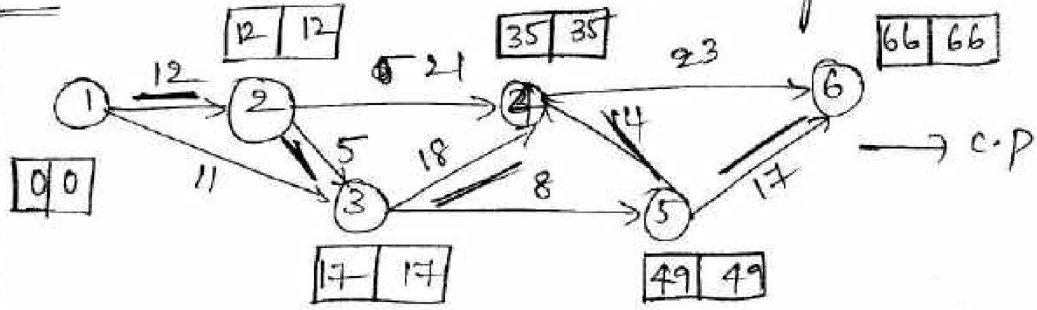
Activity	Normal Time (Tij)	Earliest time		latest time		Total float
		Earliest S.T	Earliest F.T	latest S.T	latest F.T	LS – ES

Problems:—

1. Draw a Network Diagram (or) calculate the forward processing and backward processing time (or) calculate the earliest and latest time, total flow (or) calculate crital path. and calculate the project Duration.

Activity :—	1–2	1–3	2–3	2–4	3–4	3–5	4–5	4–6	5–6
Time :—	12	11	5	21	18	8	14	23	17

Sol:— Draw the Network diagram :—



2) Forward processing time :—

$E_1 = 0$

$E_2 = E_1 + t_{ij} = 0 + 12 = 12$

$E_3 = \max(E_1 + t_{ij},\ E_2 + t_{ij}) \Rightarrow \max(0+11,\ 12+5)$

$\max(11, 17) \Rightarrow \max = 17$

$E_4 = \max(33, 35) = 35$

$E_5 = \max(E_3 + t_{ij},\ E_4 + t_{ij}) \Rightarrow \max(25, 49) = 49.$

$E_6 = \max(58, 66) = 66.$

Backward processing time:-

$L_6 = E_6 = 66.$

$L_5 = L_6 - t_{ij} = 49.$

$L_4 = \min(L_6 - t_{ij}, L_5 - t_{ij}) = \min(43, 35) = 35.$

$L_3 = \min(41, 17) = 17.$

$L_2 = \min(12, 14) = 12.$

$L_1 = \min(6, 0) = 0.$

Activity	Time (t_{ij})	Earliest timing		Latest time		Total flow
		E_s	Finishing Time	(L_s)	(L_F)	$E_s - L_s$.
1—2	12	0	0+12 = 12	0	12	$\boxed{0}$ 1-2
1—3	11	0	0+11 = 11	6	17	-6
2—3	5	12	5+12 = 17	12	17	$\boxed{0}$ 2-3
2—4	21	12	21+12 = 33	14	35	-2
3—4	18	17	18+17 = 35	17	35	$\boxed{0}$ 3-4
3—5	8	17	8+17 = 25	41	49	24
4—5	14	35	14+35 = 49	35	49	$\boxed{0}$ 4-5
4—6	23	35	23+35 = 58	43	66	-8
5—6	17	49	17+49 = 66.	49	66	$\boxed{0}$ 5-6.

Critical path:-

1-2, 2-3, 3-4, 4-5, 5-6.

1—2—3—4—5—6.

Total project duration:-

12+5+18+14+17 = 66.

Project evaluation & Review technique (PERT):-

Step-1 :- Draw the network.

Step-2 :- compute the expected duration of each activity using the formula.

Expected time $t_e = \frac{t_o + 4t_m + t_p}{6}$ & also calculate the

variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$

Step-3 :- compute the earliest times, latest times and total float of the each activity.

Step-4 :- Identify the critical activities and find the critical path.

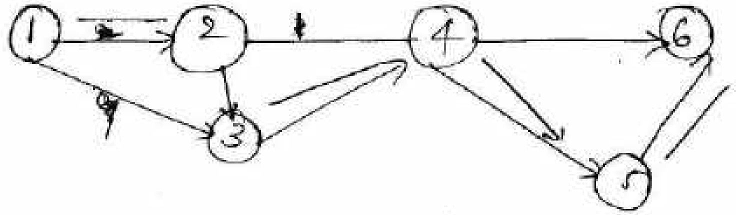
Step-5 :- compute the project length varience (σ^2) which is the sum of the varience of all the critical activities and hence find the standard deviation of the project length.

Step-6 :- calculate the standard normal varience.

$$S_D = \frac{T_S - T_E}{\sigma}$$

where T_S = Total project length (or) Total project competition.

T_E = Expected project length (or) project duration.



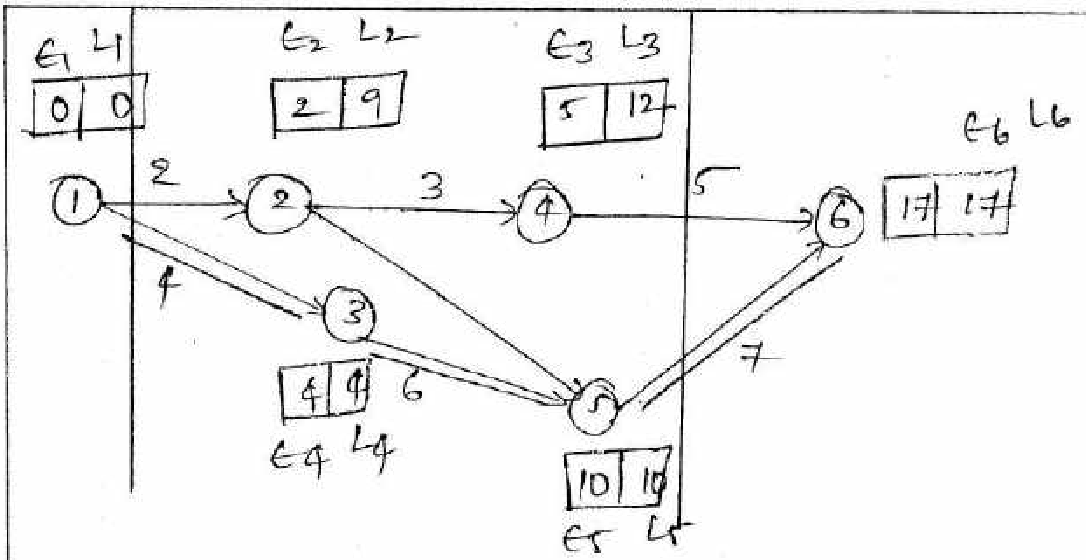
(2). A small project is composed of 7 Activities whose time is estimate are listed in the table are as follows.

Activities:	1-2	1-3	2-4	2-5	3-5	4-6	5-6.
t_0 :	1	1	2	1	2	2	3
t_m :	1	4	2	1	5	5	6
t_p :	7	7	8	1	14	8	15.

i) Draw a Network diagram, ii) Find the Expected time duration and variance of each activity iii) Variance and S.D of the project length. iv) If the project due date is 19 weeks what is the probability of making the due date.

Sol:-

Activities	Extected time $(t_e) = t_0 + 4t_m + t_p/6$.	Variance $\left(\frac{t_p - t_0}{6}\right)^2$.
1-2	$1+4(1)+7/6 = 2$	1
1-3	4	1
2-4	3	1
2-5	1	0
3-5	6	4
4-6	5	1
5-6	7	4



Critical activities:—

1—3, 3—5, 5—6.

1 — 3 — 5 — 6.

Variance $(\sigma^2) = 1+4+4 = 9$.

S.D $(\sigma) = \sqrt{9}$

$= 3$.

Total project duration $= 4+6+7 = 17$ weeks.

probability $(Z) = \dfrac{t_s - t_e / t_E}{\sigma}$

$= \dfrac{19-17}{3} = \dfrac{2}{3} = 0.66$.

$Z(0.66) = 0.7454$

If the project is performed 100 times under the same condition the probability is,

$100 \times 0.7454 = \underline{\underline{74.54}}$.

Uses of pert/ cpM (Networks) for Mgmt :-

1) The PERT/CPM techniques help the mgmt in properly planning the complicated controlling working plan and also keeping the plan upto-date. There are also helpful in searching for potential spots and in taking corrective measures.

2) The Network Techniques provide a number of checks and safeguards against going astray in developing the plan for the project. Thus there are little chances of over-sight of certain activities and events.

3) These Techniques help the mgmt in Reaching the goal with minimum time and least cost and in fore-casting the probable project duration and the associated time.

4) The Networks clearly designate the Respon-sibilities of various supervisors. The supervi-sior of a activity himself knowns the schedule precisely and also the supervisors of other activities whom be has to co-opera-te.

5) The Flexibility of the Network permits the mgmt to make the Necessary

alternation improvements as when they are needed. These allocations can be made during the development of Resources of Reviewing.

6)-Application of Network techniques has Resulted in better managerial control better utilization of Resources, Improved communication and progress Reporting, and better decision Making.

7)-Application of PERT/CPM Techniques have Resulted in saving of time which directly Results in use of cost. Also saving in time (or) early completion of the project Results in early Return of Reverse as Introduction of the competitors, Resulting in Increased profits.

Applications Areas of PERT/CPM Techniques:-

1. Building construction:- It is one of the largest areas in which the Network techniques are project mgmt have Found its Applications.

2. Administration:— Networks are used by the Administration for streamlining paper work system, in Making major Administrative changes in the system, for long Range planning and for developing plans etc.

3. Manufacturing:— The Design development and testing of New techniques, Machines, Installing Machines and plant layouts are the few examples of its applications to the Manufacturing Functions of a firm.

4. Maintenance planning:— Maintainance and shutdown of power plants, chemical plants, steel furnaces and overhauling of large Machines and PERT Techniques.

5. Research and Development:—

It is the most effective area where Pert Techniques are used for development Net used by the systems.

6. Inventory planning:— Installation of production and Inventory control,

acqusition of spare parts, etc, help in techniques.

7. Marketing:— Networks are also used for Advertising programmes for development and launching of new products for planning their distribution.

Disadvantages of Network Techniques:—

1. The difficulty arises while securing the Realistic time estimates. In the case of new and non-Respective type of projects, time estimates produced often guesses.

2. It is also some-times trouble some to develop a clear logical Network. this depends upon the data Input and produces the data.

3. The Natural tendency to oppose changed Results in the difficulty of persauding the mgmt to accept.

4. Determination to the level of Network details judgement and Experience.

5. The planning and Implementation of Networks technology Recruit trained Personnel.